NOTES

ON

STEREOTOMY

NOTES ON STEREOTOMY,

PREPARED FOR THE USE OF STUDENTS

IN

CIVIL ENGINEERING

IN THE

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,

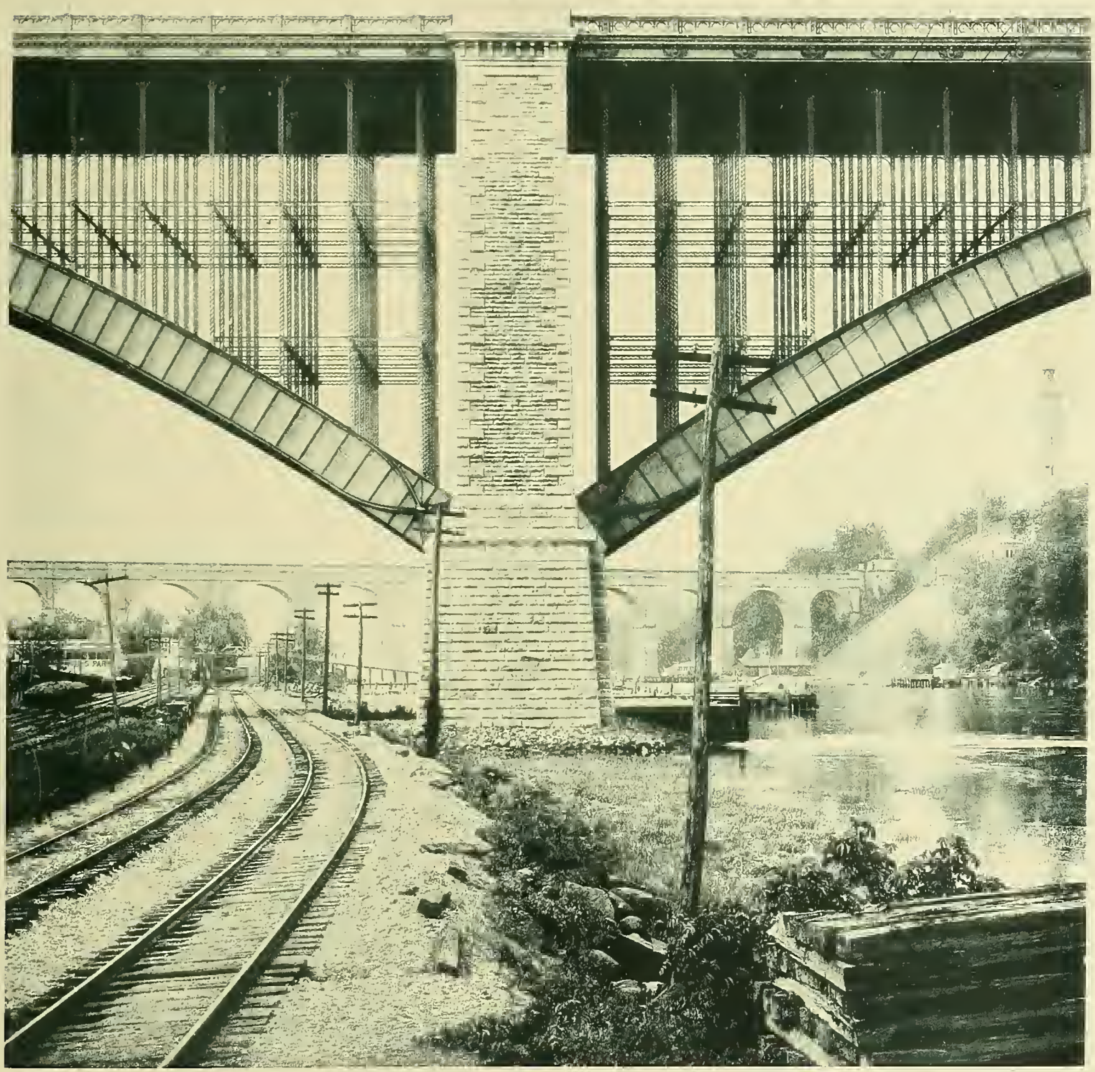
BY

DWIGHT PORTER.

PLANE, SINGLE-CURVED AND DOUBLE-CURVED SURFACES.

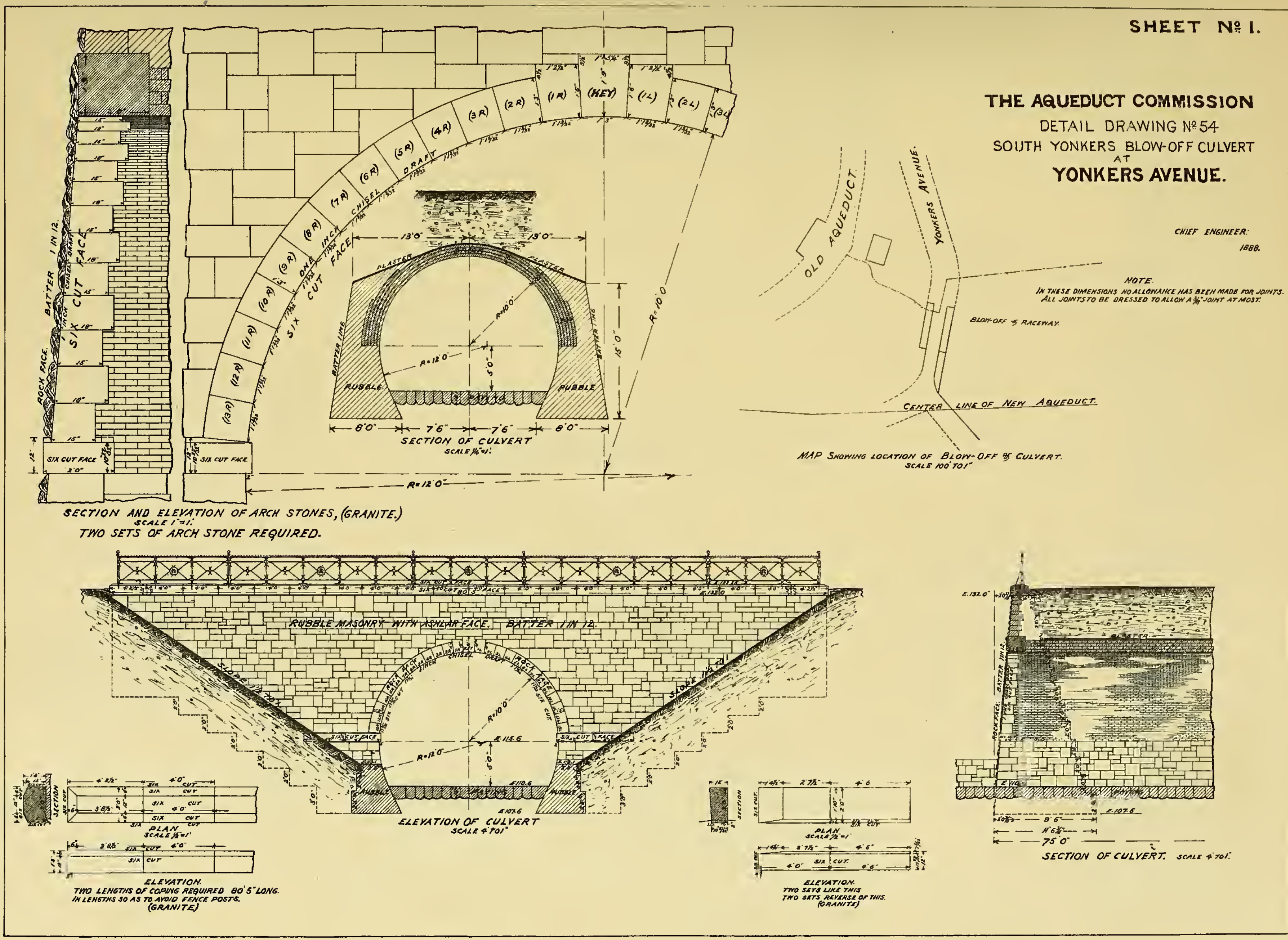
85

1916



Geo. H Walker & Co., Boston.

CENTRAL PIER, WASHINGTON BRIDGE, NEW YORK.



SHEET No. I.
THE AQUEDUCT COMMISSION
DETAIL DRAWING No. 54
SOUTH YONKERS BLOW-OFF CULVERT
AT
YONKERS AVENUE.
CHIEF ENGINEER:
1888.
NOTE.
IN THESE DIMENSIONS NO ALLOWANCE HAS BEEN MADE FOR JOINTS.
ALL JOINTS TO BE DRESSED TO ALLOW A 3/8" JOINT AT MOST.
OLD AQUEDUCT.
YONKERS AVENUE.
BLOW-OFF & RACEWAY.
CENTER LINE OF NEW AQUEDUCT.
MAP SHOWING LOCATION OF BLOW-OFF & CULVERT.
SCALE 100' TO 1"
SECTION OF CULVERT
SCALE 1/8"=1'.
SECTION AND ELEVATION OF ARCH STONES, (GRANITE.)
SCALE 1"=1'.
TWO SETS OF ARCH STONE REQUIRED.
SIX CUT FACE
ONE INCH CHISEL DRAFT
ROCK FACE.
BATTER 1 IN 12.
RUBBLE
PLASTER
BATTER LINE
R=10'0"
R=12'0"
RUBBLE MASONRY WITH ASHLAR FACE. BATTER 1 IN 12.
SLOPE 1 1/2 TO 1
ELEVATION OF CULVERT
SCALE 4' TO 1"
SECTION OF CULVERT. SCALE 4' TO 1'.
E.132.0'
E.115.6
E.110.6
E.107.6
75'0"
PLAN
SCALE 1/2"=1'
SECTION
ELEVATION.
TWO LENGTHS OF COPING REQUIRED 80'5" LONG.
IN LENGTHS SO AS TO AVOID FENCE POSTS.
(GRANITE)
ELEVATION.
TWO SETS LIKE THIS
TWO SETS REVERSE OF THIS.
(GRANITE)

SHEET Nº 11

THE AQUEDUCT COMMISSION
DETAIL DRAWING Nº 7,
OF MASONRY
FOR
POCANTICO BLOW-OFF
AND
WASTE WEIR.

SCALE ½ IN = 1 FT

Plan of top of Course XI
Rise 22 inches.
Ele. 139. 25

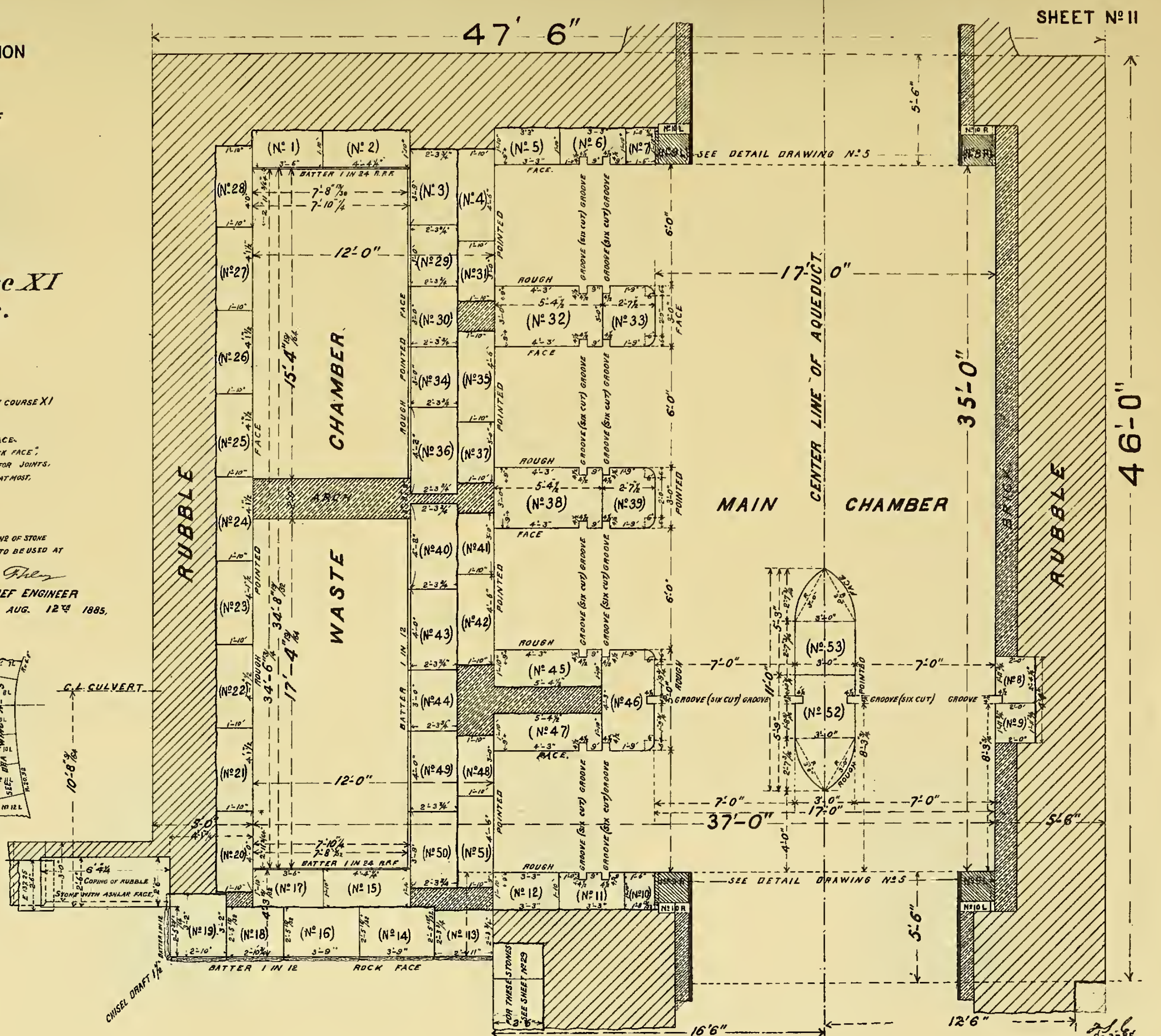
NOTE.

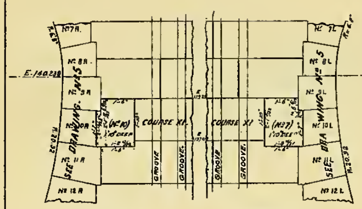
ALL EXPOSED FACES INSIDE OF CHAMBER SHOWN ON COURSE XI TO BE "ROUGH POINTED" FACE
EXCEPTING
ALL GROOVES WHICH ARE TO BE "SIX CUT" FACE.
ALL EXPOSED FACES OUTSIDE OF STRUCTURE TO BE "ROCK FACE".
IN THESE DIMENSIONS NO ALLOWANCE HAS BEEN MADE FOR JOINTS.
ALL STONES TO BE DRESSED TO ALLOW A ⅜" JOINT AT MOST.

REMARKS.

IN MARKING THE STONE FOR SHIPMENT PREFIX TO Nº OF STONE THE LETTERS P.D. INDICATING THAT THE STONE IS TO BE USED AT POCANTICO

A. Fteley
DEPTY CHIEF ENGINEER
AUG. 12TH 1885.

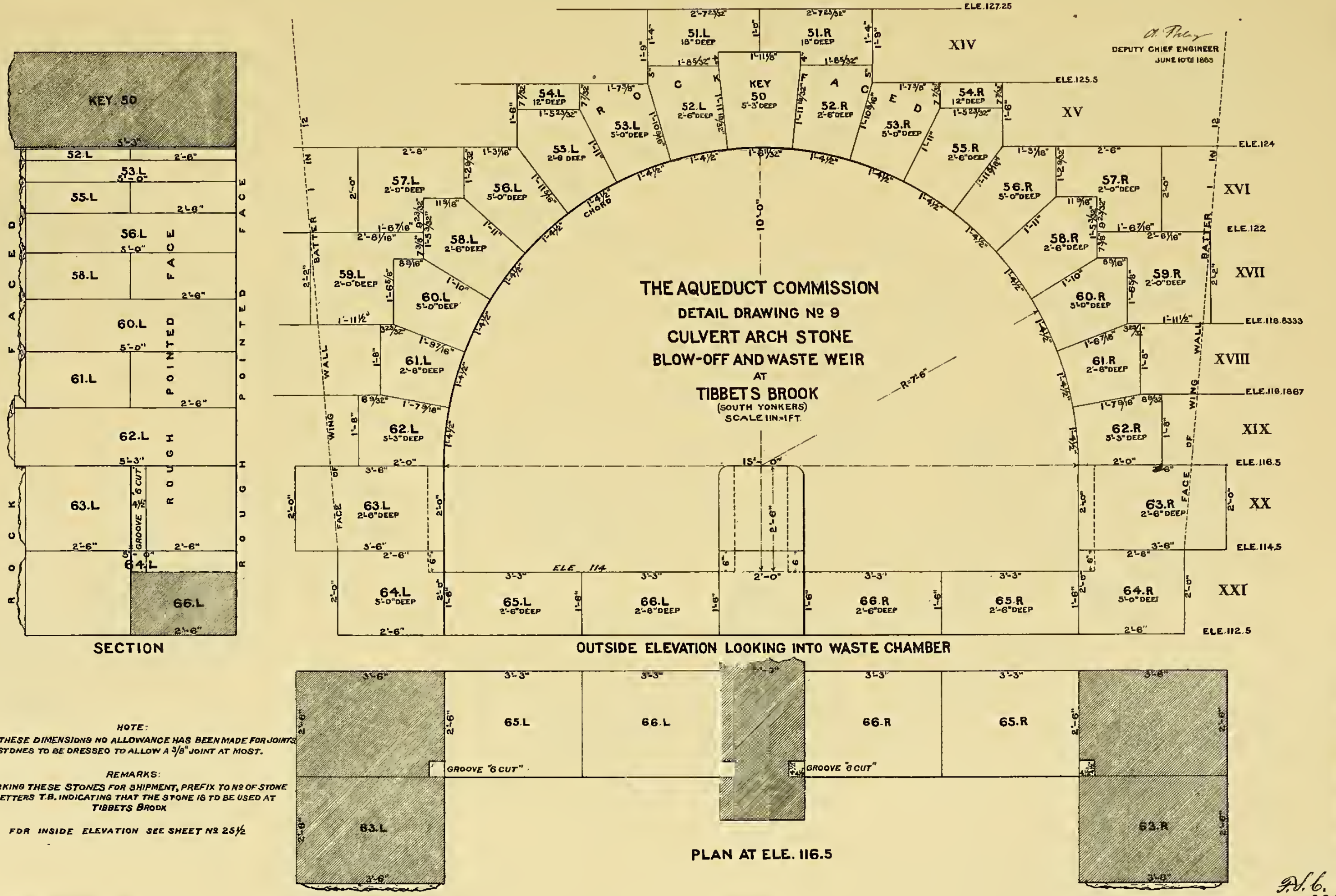




ELEVATION AT SOUTH PORTAL.

ELEVATION AT NORTH PORTAL

SHEET Nº 25



NOTE:
IN THESE DIMENSIONS NO ALLOWANCE HAS BEEN MADE FOR JOINTS.
ALL STONES TO BE DRESSED TO ALLOW A 3/8" JOINT AT MOST.

REMARKS:
IN MARKING THESE STONES FOR SHIPMENT, PREFIX TO Nº OF STONE THE LETTERS T.B. INDICATING THAT THE STONE IS TO BE USED AT TIBBETS BROOK

FOR INSIDE ELEVATION SEE SHEET Nº 25½

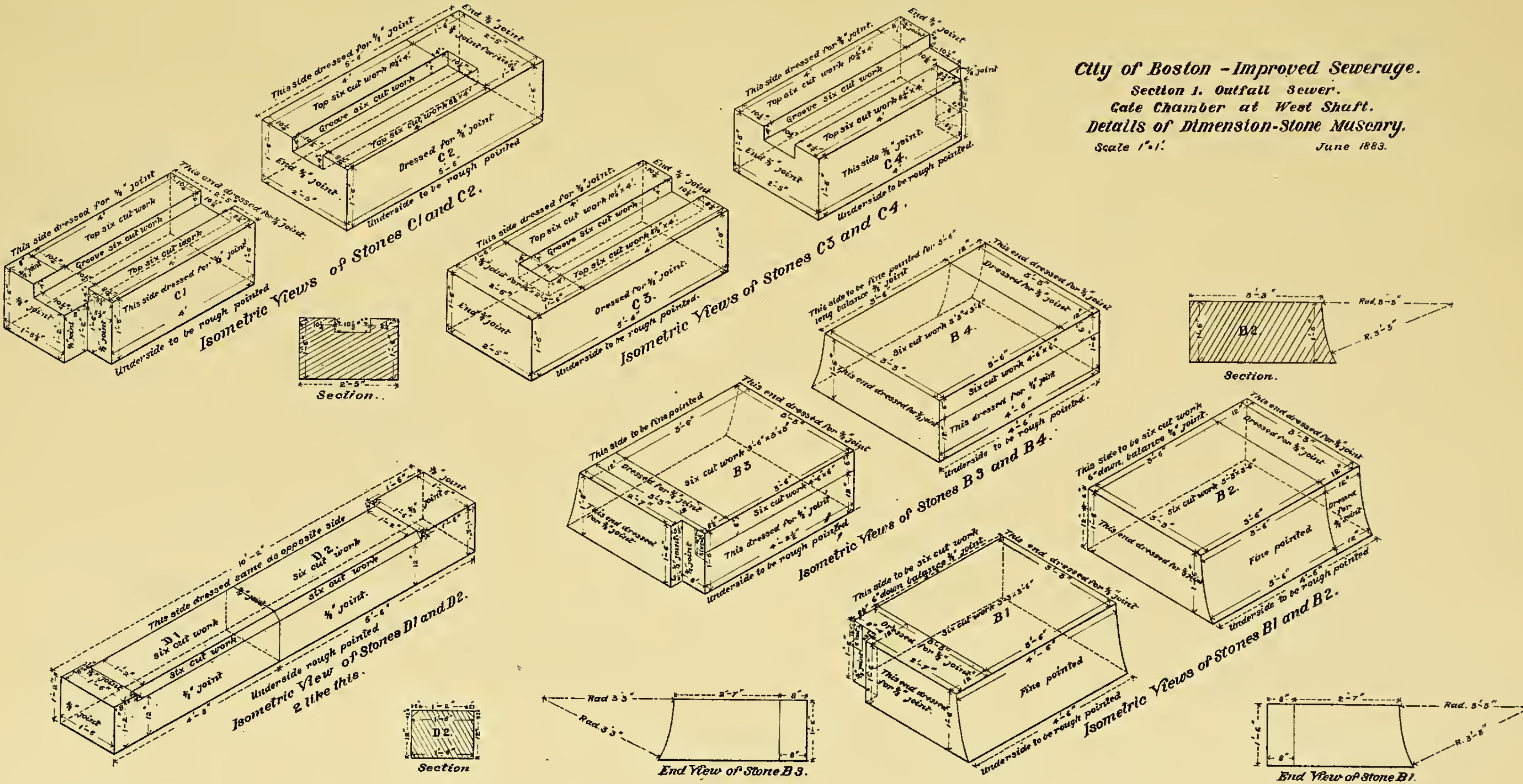
City of Boston - Improved Sewerage.

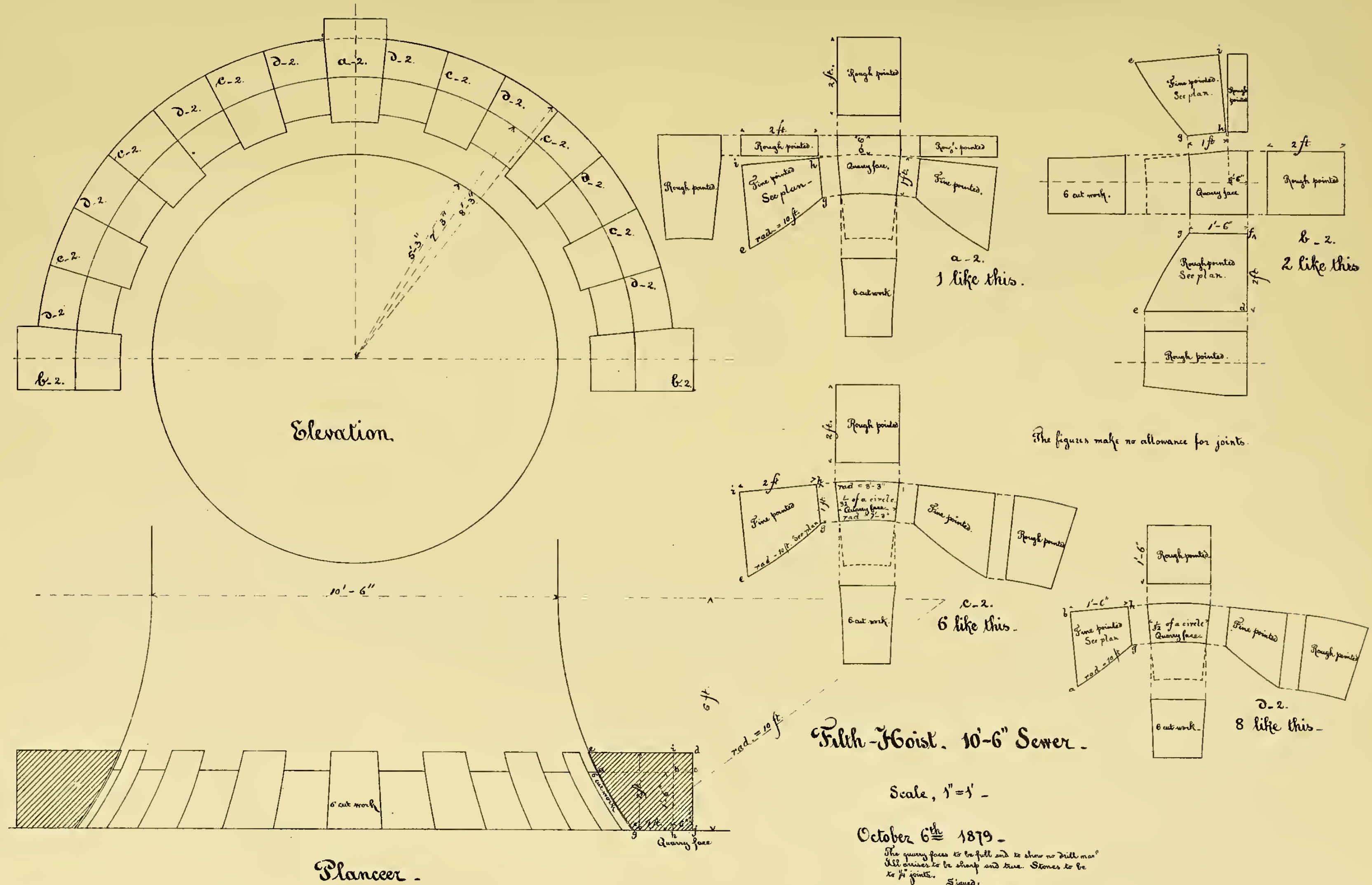
Section 1. Outfall Sewer.

Gate Chamber at West Shaft.

Details of Dimension-Stone Masonry.

Scale 1"=1'. June 1883.





Filth-Hoist. 10'-6" Sewer.

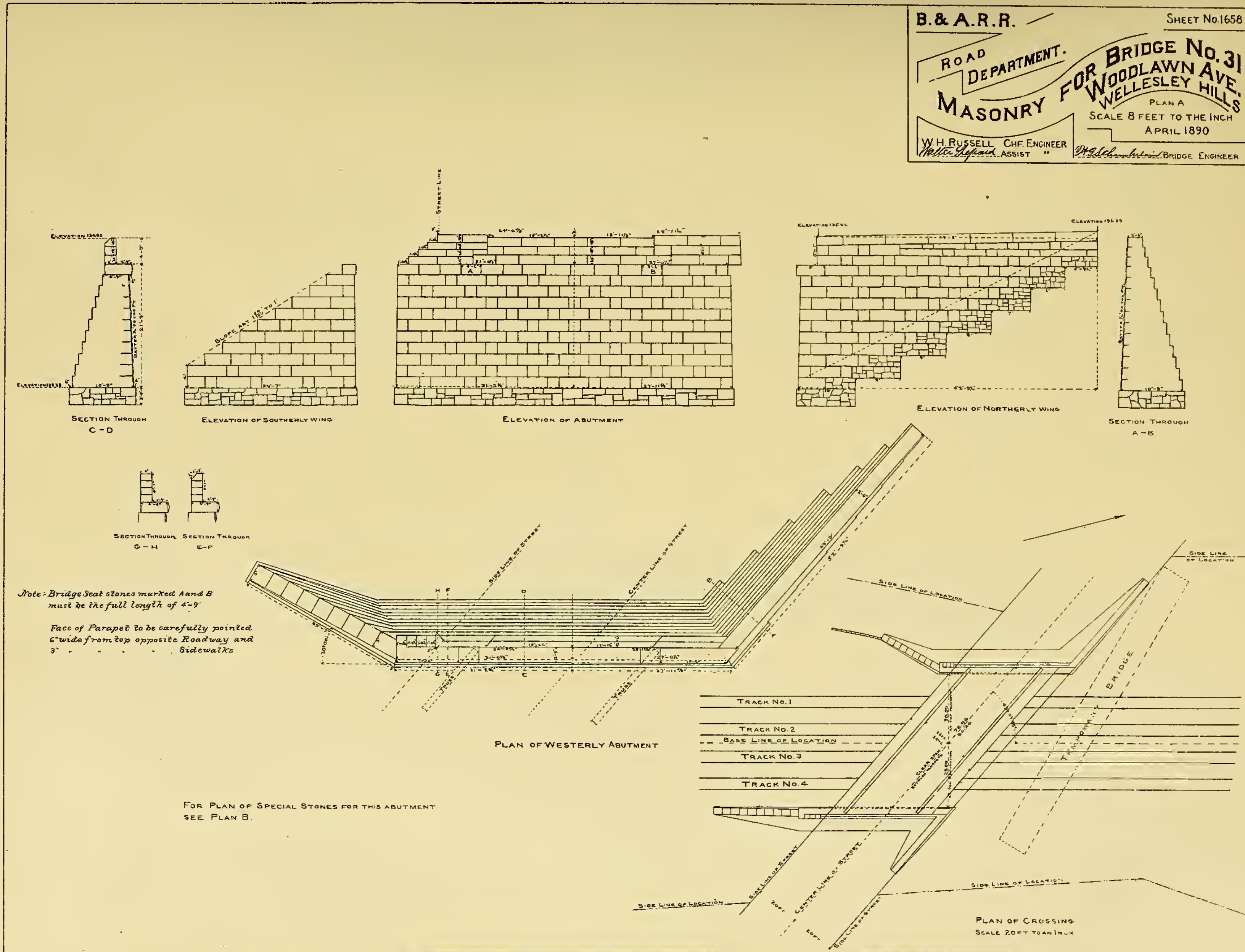
Scale, 1" = 1'.

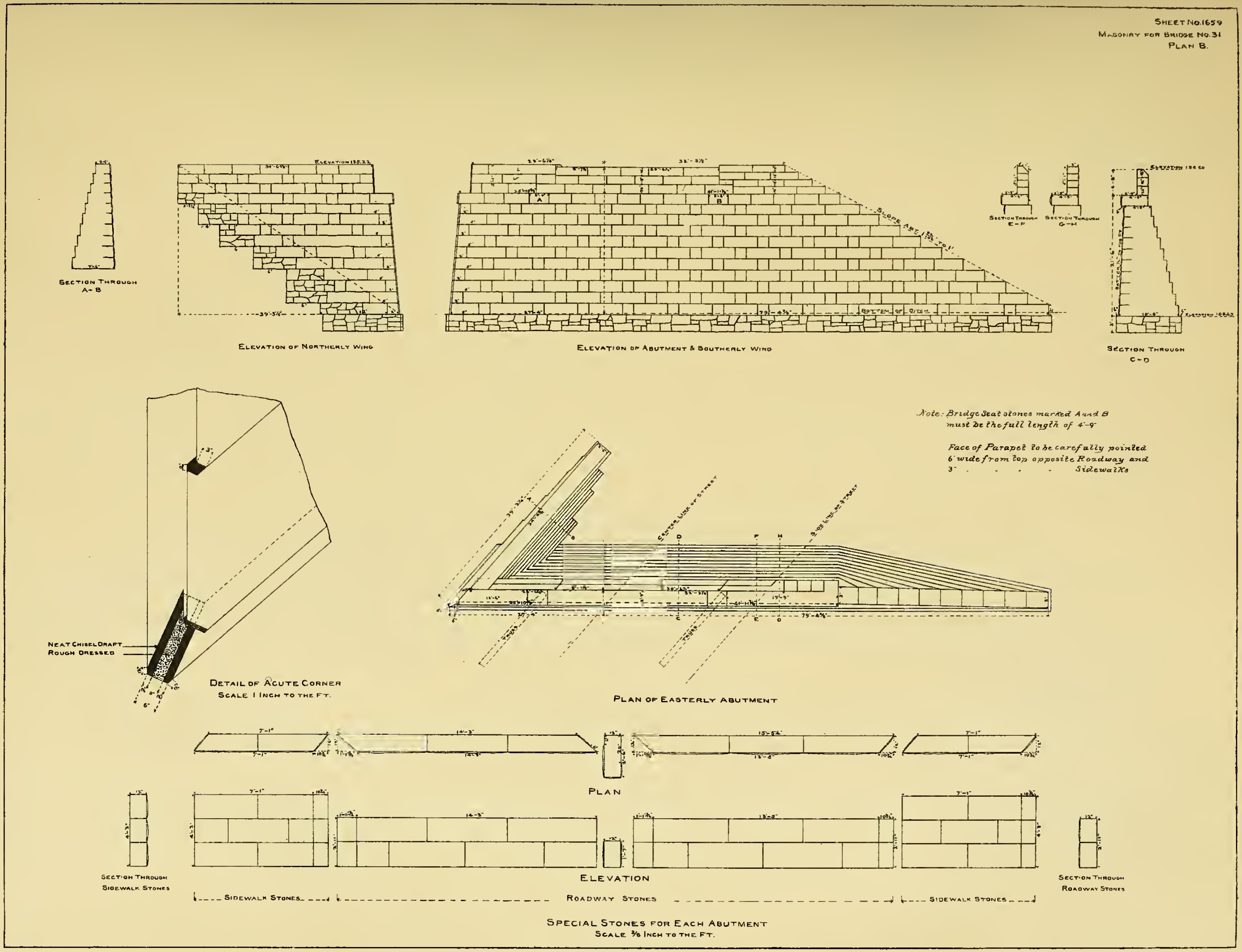
October 6th 1879.

The quarry faces to be full and to show no drill mar[ks].
All arrises to be sharp and true. Stones to be to 1/4" joints.

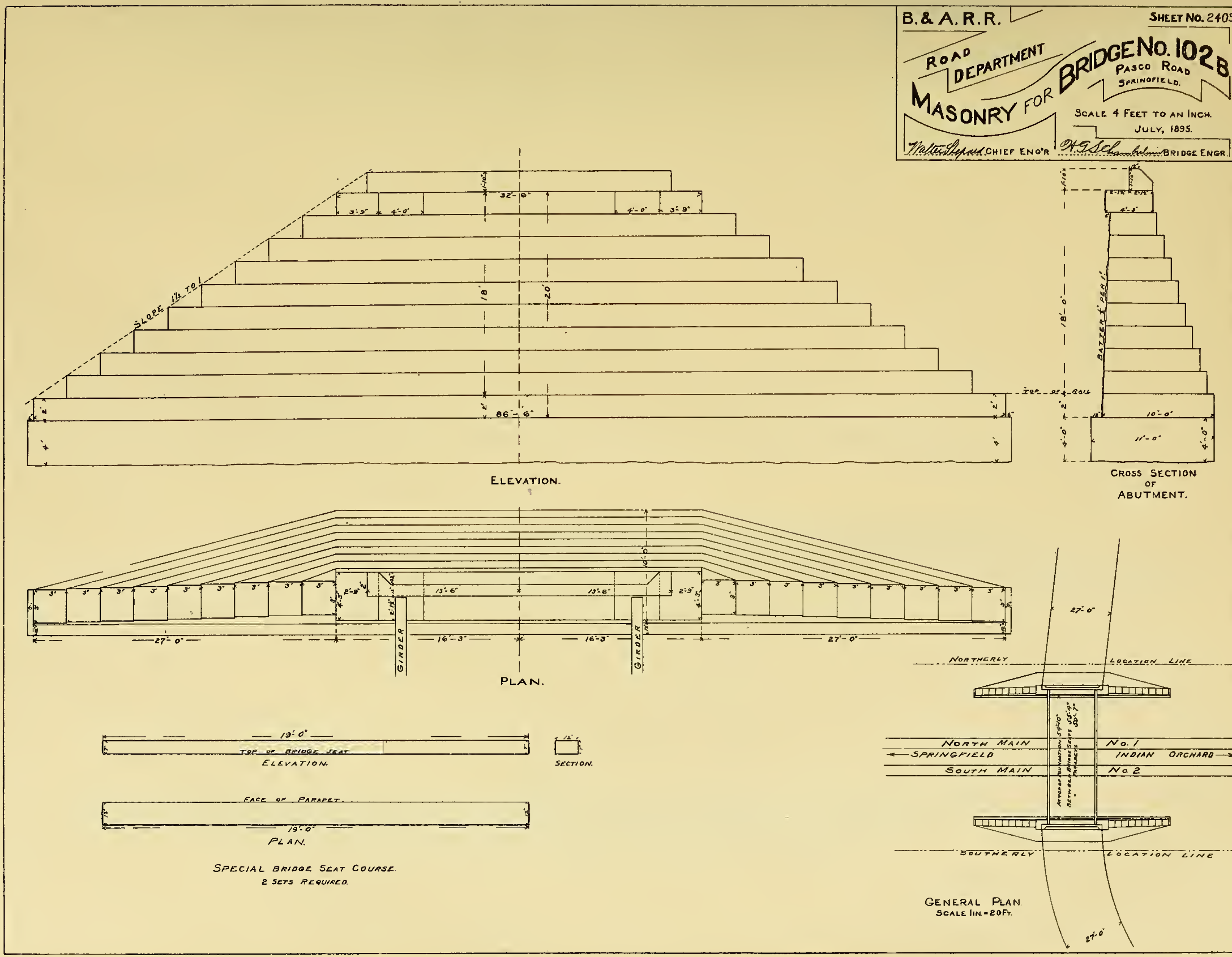
Signed: Jas. P. Davis,
City Engineer

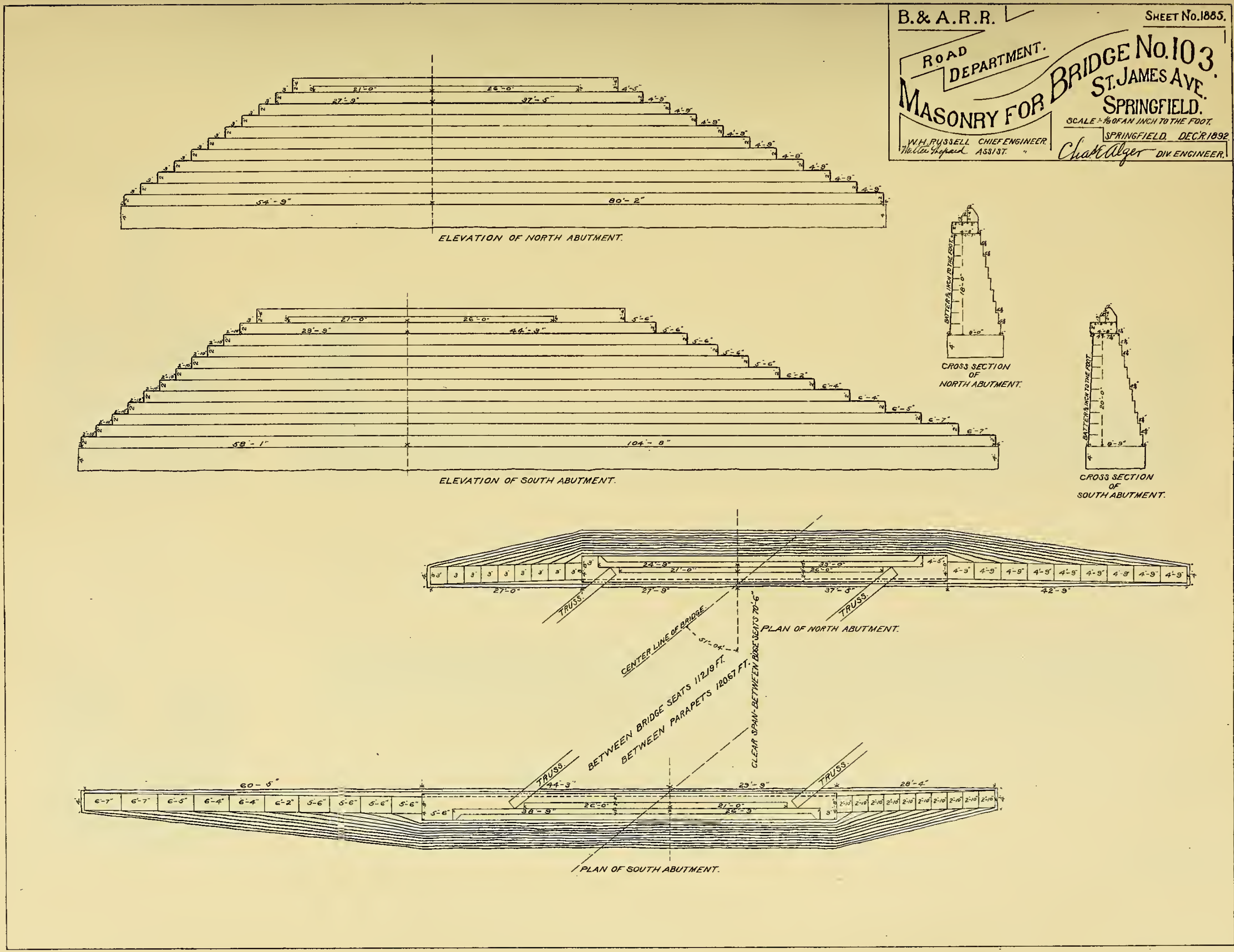
Nov. 1, 1879.

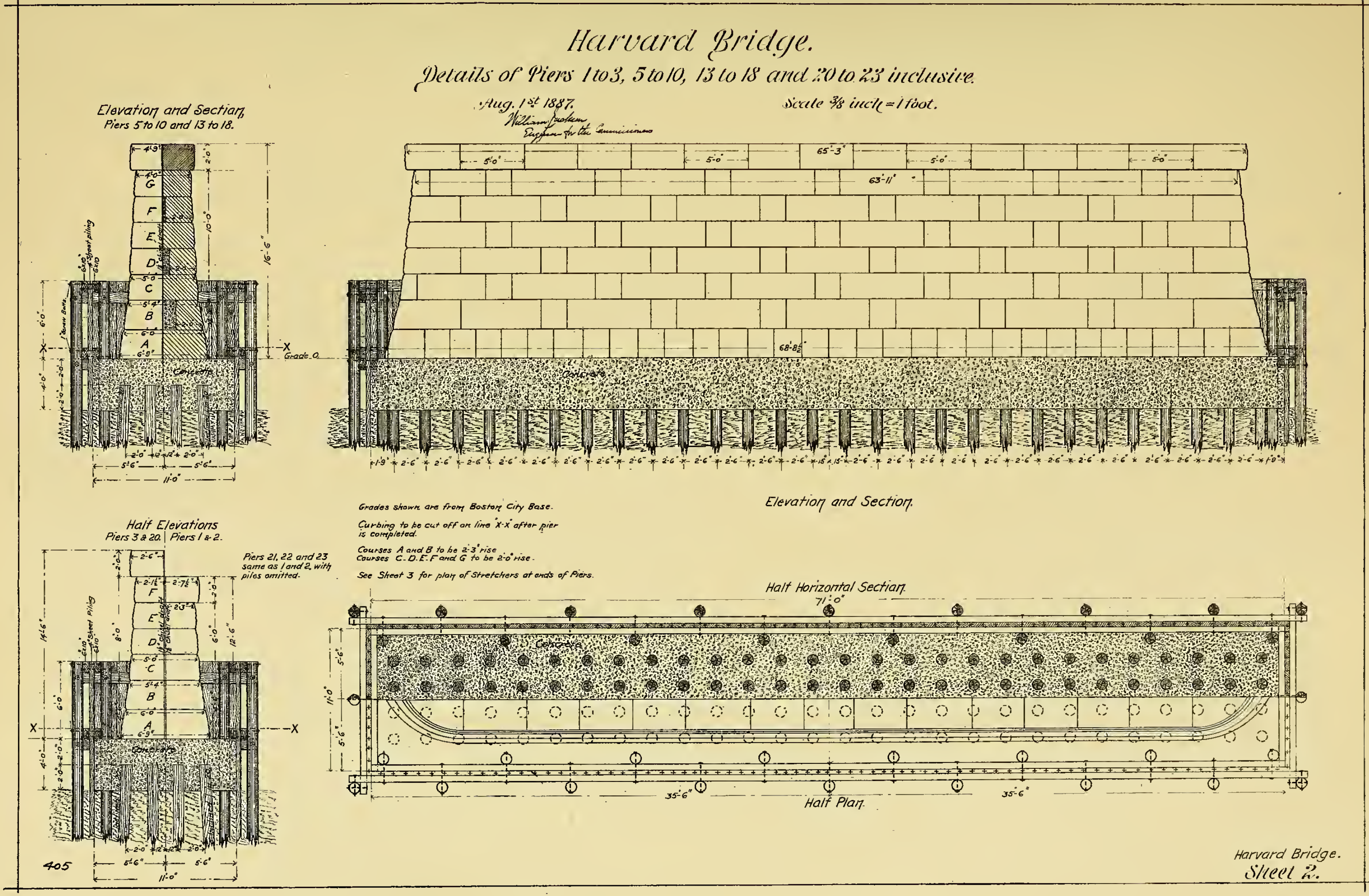

B. & A.R.R.
Sheet No. 1658
Road Department.
Masonry for Bridge No. 31
Woodlawn Ave. Wellesley Hills
Plan A
Scale 8 feet to the inch
April 1890
W. H. Russell Chf. Engineer
Walter Shepard Assist "
Bridge Engineer
Street Line
Section Through C – D
Elevation of Southerly Wing
Elevation of Abutment
Elevation of Northerly Wing
Section Through A – B
Section Through G – H
Section Through E – F
Note: Bridge Seat stones marked A and B must be the full length of 4'-9"
Face of Parapet to be carefully pointed 6" wide from top opposite Roadway and 3" " " " " Sidewalks
Side Line of Street
Center Line of Street
Plan of Westerly Abutment
For Plan of Special Stones for this abutment see Plan B.
Side Line of Location
Bridge
Track No. 1
Track No. 2
Base Line of Location
Track No. 3
Track No. 4
Center Line of Street
Plan of Crossing
Scale 20 ft to an inch

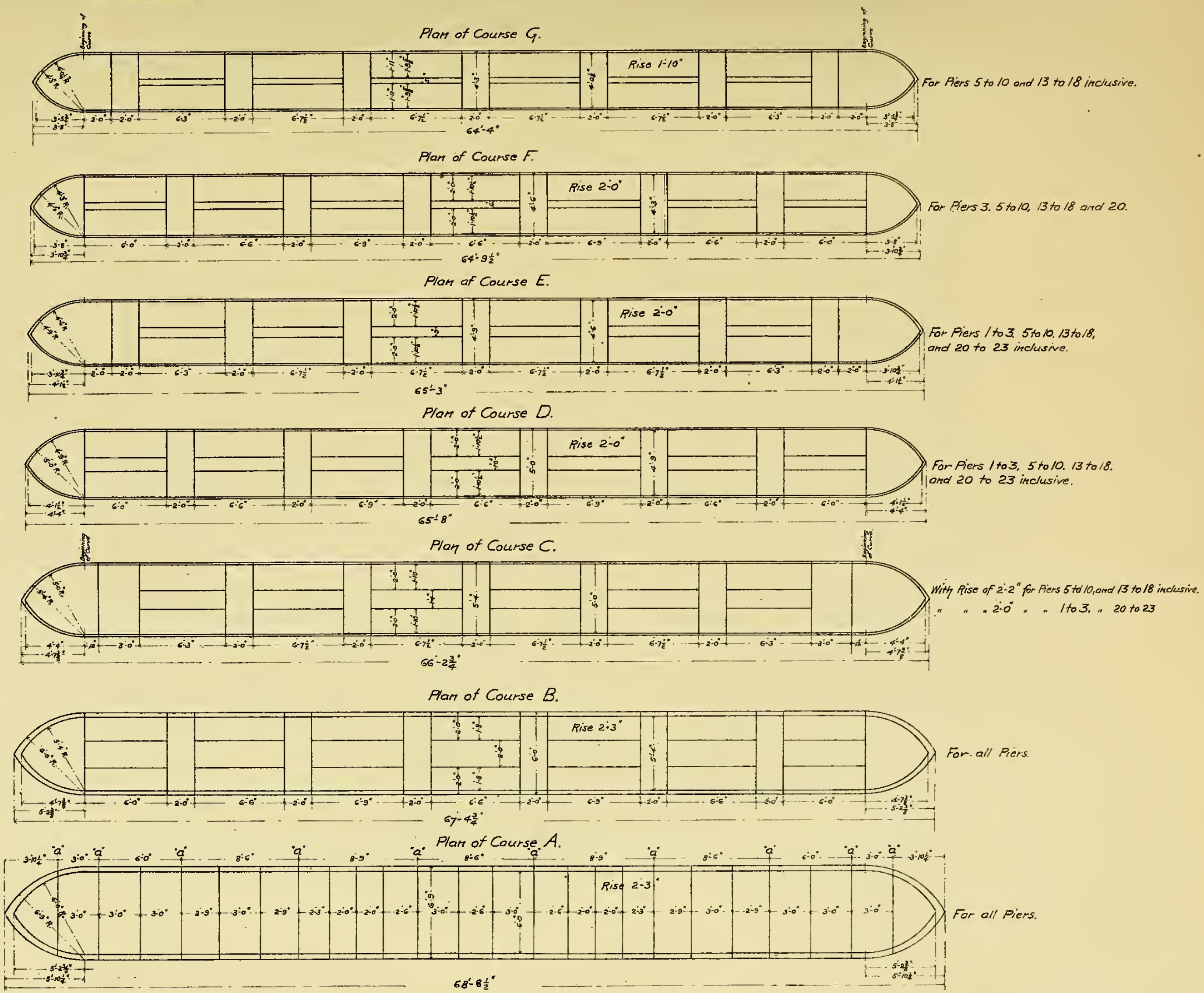


Sheet No.1659
Masonry for Bridge No.31
Plan B.
Section Through A-B
Elevation of Northerly Wing
Elevation 138.22
39'-5¼"
Elevation of Abutment & Southerly Wing
Slope Abt. 1¾ to 1
Bottom of Ditch
79'-4½"
Section Through E-F
Section Through G-H
Section Through C-D
Note: Bridge Seat stones marked A and B must be the full length of 4'-9"
Face of Parapet to be carefully pointed 6" wide from top opposite Roadway and 3" . . . . Sidewalks
Center Line of Street
Neat Chisel Draft
Rough Dressed
Detail of Acute Corner
Scale 1 Inch to the Ft.
Plan of Easterly Abutment
Plan
Elevation
Section Through Sidewalk Stones
Section Through Roadway Stones
Sidewalk Stones
Roadway Stones
Sidewalk Stones
Special Stones for Each Abutment
Scale ⅜ Inch to the Ft.


B. & A. R.R.
Sheet No. 2409
Road Department
Masonry for Bridge No. 102 B
Pasco Road
Springfield.
Scale 4 Feet to an Inch.
July, 1895.
Chief Eng'r
Bridge Engr.
Elevation.
Slope 1½ to 1
32'-6"
86'-6"
18'
20'
Cross Section of Abutment.
Batter 1" per 1'
18'-0"
10'-0"
11'-0"
Plan.
Girder
Girder
27'-0"
16'-3"
16'-3"
27'-0"
13'-6"
13'-6"
19'-0"
Top of Bridge Seat
Elevation.
Section.
Face of Parapet
19'-0"
Plan.
Special Bridge Seat Course.
2 Sets Required.
Northerly Location Line
North Main No. 1
Springfield Indian Orchard
South Main No 2
Southerly Location Line
General Plan.
Scale 1in.=20Ft.
27'-0"


B. & A. R. R.
SHEET No. 1885.
ROAD DEPARTMENT.
MASONRY FOR BRIDGE No. 103.
ST. JAMES AVE.
SPRINGFIELD.
SCALE: 1/8 OF AN INCH TO THE FOOT.
SPRINGFIELD. DEC'R 1892.
W. H. RUSSELL CHIEF ENGINEER.
Walter Shepard ASSIST. "
Chas. A. Alger DIV. ENGINEER.
ELEVATION OF NORTH ABUTMENT.
ELEVATION OF SOUTH ABUTMENT.
CROSS SECTION OF NORTH ABUTMENT.
CROSS SECTION OF SOUTH ABUTMENT.
BATTER ½ INCH TO THE FOOT
PLAN OF NORTH ABUTMENT.
PLAN OF SOUTH ABUTMENT.
TRUSS.
CENTER LINE OF BRIDGE.
BETWEEN BRIDGE SEATS 112.19 FT.
BETWEEN PARAPETS 120.67 FT.
CLEAR SPAN-BETWEEN BASE SEATS 70'-6"


Harvard Bridge.
Details of Piers 1 to 3, 5 to 10, 13 to 18 and 20 to 23 inclusive.
Aug. 1st 1887.
William Jackson
Engineer for the Commissioners
Scale 3/8 inch = 1 foot.
Elevation and Section
Piers 5 to 10 and 13 to 18.
Half Elevations
Piers 3 & 20. | Piers 1 & 2.
Piers 21, 22 and 23 same as 1 and 2, with piles omitted.
Grade 0
Concrete
Sheet Piling
Elevation and Section.
Grades shown are from Boston City Base.
Curbing to be cut off on line "X-X" after pier is completed.
Courses A and B to be 2'-3" rise
Courses C, D, E, F and G to be 2'-0" rise.
See Sheet 3 for plan of Stretchers at ends of Piers.
Half Horizontal Section.
71'-0"
Half Plan.
35'-6"
65'-3"
63'-11"
68'-8½"
Harvard Bridge.
Sheet 2.



Harvard Bridge.

Courses in Piers.

Scale 3/8 inch = 1 foot.

Dec. 1887.

NOTE. The joints marked "a" must be in the positions shown; the others may be varied.

This plan revised March 2, 1888 at the suggestion of the Contractors.

STEREOTOMY.

1. Introductory. Stereotomy is defined as "the science or art of cutting solids into certain figures or sections." It thus includes, and the term is sometimes used as synonymous with, the art of cutting stones for masonry structures. This application only will here be considered. Full knowledge of all the minor details of practical stone-cutting is not essential for the engineer. He should, however, be competent to prepare whatever drawings and patterns may be required in any masonry construction, and should understand the mode of procedure to be followed in bringing the stones to shape. The proportions of the structure are here assumed to have been fixed by the designer, with reference to the laws of stability and of architecture, and the study to be undertaken deals simply with the sub-division into stones, and the preparation of the necessary general and detail drawings. Most of the problems here presented, with their accompanying plates, have been taken from *Traité Pratique de la Coupe des Pierres, par Émile Lejeune.*

2. Drawings. The number and character of drawings required for a stone structure evidently must be governed by circumstances. Ordinarily there must be a plan, an elevation and a section, at least. If the structure is complicated the drawings become correspondingly numerous, and comprise elevations from various points of view, together with sundry plans, sections and other details. If definite sizes of stones are to be required, and a particular arrangement in the structure, course plans showing such arrangement in detail must be prepared.

General drawings should commonly be made, if practicable, to as large a scale as one-half or three-quarters of an inch to the foot; detail drawings to as large a scale as is necessary for clearness, and frequently they must be of full size.

All dimensions for stone-cutters and masons are to be expressed in feet, inches, halves, quarters, etc., to the nearest 1/4 or 1/8 of an inch, and in some cases to the nearest 1/16 or 1/32 of an inch.

If a stone is to be rectangular in shape, a simple sketch with dimensions may be sufficient guide to the stone-cutter; but if irregular, sundry sketches, an isometric view, etc., may be needed. Patterns (moulds) and templets made of wood, zinc or other material, to correspond to various faces, sections or profiles, may also be required. To prepare these, full-size drawings of the structure are made on a large floor or other surface. For structures of moderate size the instruments needed for such drawings are mainly large beam compasses, a long wooden straight-edge and a large steel square; but for the largest drawings it may be necessary to use cords or wires in determining points or lines. If the stone is unusually complicated in shape or involves carving, a full-size or half-size model may be necessary. For such work as an emblematic design, the original designer first prepares a sketch; the sculptor then constructs a clay model, using the sketch as a basis, and generally modifying it somewhat according to his own taste; from the clay model a plaster cast is made, and from this the stone-cutter works, transferring points from cast to stone as frequently and with as great accuracy as the case demands.

Drawings for masonry are commonly made without reference to the thickness of joints, and where necessary in order to prevent misunderstanding, especially in the case of detail drawings, a note should be added to the effect that "in this drawing no allowance has been made for joints." The thickness assigned on the drawing to any course is interpreted by the stone-cutter as the distance from the top of one stone to the top of the next lower one, as laid. (See Fig. 1)

The drawings must give all necessary dimensions, and must also make it entirely clear which surfaces are to be exterior and visible when the stone is in place and which are to be joint surfaces, the style of finish to be given the faces, and the thickness to be assumed for joints. The date of the drawing, and in some cases the name or initials of the draftsman, should also be added. In dimension work a system of numbering courses and stones, for convenience in subsequent identification, is generally adopted. The accompanying plates, reduced from the original drawings, will serve to illustrate some of the features to which reference has been made.

In any drawing discretion must be exercised as to what lines or portions of the structure it is best to show in that particular view, and what to omit, and the draftsman should not allow himself to be hedged in by unvarying rules in this matter. The object should be to make it clear and unmistakable what general form or what details, as the case may be, are proposed for the structure in question. Lines which in any particular view are necessary to this end should be given; those which are not, or which serve to confuse the drawing, or which can be better shown in another view, may be omitted. The following principles are commended to the notice of the student (See "Practical Hints for Draftsmen," MacCord):-

(a) "In each separate view whatever is shown at all should be represented in the most explanatory manner."
(b) "That which is not explanatory, in any one view, may be omitted therefrom, if sufficiently defined in other views."
(c) "The proper position of a cutting plane is that by which the most information can be clearly given."
(d) "It is not necessary to show in section everything which might be divided by the cutting plane."
(e) "Whatever lies beyond a cutting plane may be omitted when no necessary information would be conveyed by its representation."

A section or development may be made on a broken, or even upon a curved line, if anything is to be gained by so doing. For a figure symmetrical about a centre line, it is often sufficient, except for a picture drawing, to show only one half in any one view, a half plan or half elevation sometimes being combined with a half section.

So far as making the drawings is alone concerned, Stereotomy is for the student an application of Descriptive Geometry, the problems most frequently arising relating to the intersection and development of surfaces. When accurate execution is required the student should be very careful in laying out the main lines of his drawing, particularly as regards dimensions, parallels and perpendiculars. Economy of labor and quickness of work must constantly be aimed at and, above all, the greatest care must be taken in accurately and clearly noting dimensions on the drawing. The directions given below will aid in obtaining satisfactory results:-

(a) In laying off by scale a number of successive distances along a line, lay off the successive sums of the distances, referred in each case to the same starting point, rather than lay off one distance after another, referring each time to the point last fixed.
(b) In laying off a distance too short to be easily given by the dividers, set back from the reference point any arbitrary distance, and then set forward from the point thus fixed the assumed distance plus the one required.
(c) To draw a parallel to a given line, when the T square or triangles are not applicable, strike arcs at the extremities of the given line and make the required line tangent to the arcs.
(d) Lines which are to determine a point by their intersection should preferably not meet at a very acute angle, and points which are to fix a line should preferably be chosen at a distance apart greater than the required length of line.
(e) The drawing should be carefully verified by the draftsman, and advantage taken of all convenient methods of testing its accuracy.
(f) The pencilling of the drawing should be made complete and accurate before inking is begun.

3. Tracings on Cloth. The usual purpose of a tracing on cloth is to permit the duplication of drawings by the blue process. Fine lines should therefore be avoided and for the colors of lines only black or vermilion should be used. The student will find the rough side of the cloth the more satisfactory to use, and if, on account of slight greasiness, it does not take ink well, the trouble may be remedied by first dusting the surface with powdered chalk, which is to be brushed off before inking. Erasures may be made with a sharp knife blade, taking care not to gouge into the cloth.

4. Reduction of Drawings by Photography. If drawings are to be reduced by photography care must be taken that the lines, letters and figures in the original are coarse enough to endure the proposed reduction without becoming indistinct. With this precaution observed, the general appearance of a drawing will be improved by reduction, owing to the greater sharpness of the lines and the disappearance of minor imperfections. If dimensions are to be scaled from the reduced copy a scale evidently must be constructed upon the original sheet which shall be reduced in the same proportion as the drawing itself. As regards the lettering of drawings for photo-reproduction see an article by Charles W. Reinhardt in Engineering News of June 13, 1895.

To avoid the inconvenience of handling large blue prints in the field or shop, copies of the original drawing or tracing, reduced by photography, are sometimes used. These may to advantage be mounted upon cardboard and varnished. So-called "black prints" can be successfully copied and reduced by photography. A method of copying, to the number of forty or more duplicates, drawings made in colored lines or washes, the colors being reproduced in the copies, is described in Engineering News, Aug. 24, 1893, p.160.

5. Proportions and Shape of Stones. The depths to be given the various courses and the proportions to be observed in the dimensions of the stones, are determined with regard to strength, appearance and economy. Tolerably large stones, properly arranged, bind the whole work well together and tend to secure uniform distribution of pressure. No commensurate advantage is ordinarily to be gained, however, from excessive size, while on the other hand greatly increased difficulty and cost in handling are involved, and there is also danger of the cracking of very long stones, owing to the stresses from unequal settlement in the mass of masonry. By requiring courses to be of uniform depth and the stones to be of uniform size, an appearance of great regularity may be gained for the face of the structure; but to insist upon such uniformity imposes upon the quarryman limitations which materially increase the cost of the work. Economy will therefore result from allowing, when possible, a moderate range in thickness of courses, as well as in other dimensions of the stones. A variation in depth of courses, these being arranged to diminish regularly in thickness from bottom to top of wall, is also in accordance with good taste.

In engineering structures the common range in thickness of courses is from 10 or 12 up to 30 inches. Stones are usually required to be at least one and one-fourth or even one and one-half times as wide as deep. They are allowed a length as great as from three to six times the depth, varying accordingly as the material is weak or strong. In copings, parapets and other special features of structures, where the stones are not exposed to heavy pressure from above, it is not essential, however, to adhere to the proportions which have been indicated.

The bed joints of a stone should be made normal, or nearly so, to the pressure to be sustained; the other joints should be arranged so far as possible to produce right angles and to give the stone a simple form. Acute angles are objectionable because they are likely to lead to the cracking of the stone. Two distinct bearing surfaces for the same stone should also be avoided, since in case of poor fitting and consequent unequal bearing there is danger of cracking.

6. Thickness of Joints. The thickness of joints should be consistent with the general standard aimed at in the masonry, being properly least for work which is to be of the highest class as regards strength and finish. The requirement of thin joints involves a nicer dressing of the joint surfaces of the stones than would otherwise be given.

In engineering structures presenting the best type of work, such as arches, waterways, piers, etc., the joints are made from one-fourth inch to one-half inch thick. In heavy work of a less high grade the joints range from one-half inch to one inch in thickness. In fine work in building construction joints as thin as one-eighth inch or even less are employed. In brick work of a good class the mortar joints are from one-eighth inch to three-eighths inch in thickness, and in coarser work one-half inch is considered allowable.

Since the bed-joint surfaces of the stones receive and transmit the downward pressures, they should be dressed substantially true throughout. There is not the same necessity for this, however, with the vertical joint surfaces, and frequently specifications allow a moderate increase in thickness of vertical joints at a certain distance, say 12 inches, back from the face of the wall, and even make a distinction as to thick-

ness between the two sets of joints at the face of the structure. In speaking of the thickness of a joint, either the space or the mortar filling between two adjacent stones is evidently in mind; but in practice the term joint is also applied to the adjacent surfaces of the stones, when these are not bearing surfaces, while bearing surfaces are termed beds.

7. Modes of Bonding. It is important that the stones should be so arranged in any masonry structure as to tie the whole mass together laterally, and at the same time cause the various downward pressures to be evenly distributed toward and upon the foundation. This is accomplished in part by avoiding continuous vertical joints fron course to course and, indeed, by separating or breaking such joints as widely as possible; and, farther, by the use of headers and stretchers. (Fig.2) Stretchers are placed with their longest dimension showing in the face of the wall; while headers (also called binders and throughs) have their longest dimension running transversely through the wall.

Headers should be evenly distributed through the mass of masonry, and to advantage headers and stretchers may regularly alternate in each course, the headers in one course being arranged so far as practicable over the middle of stretchers in the course below, or at least so as to break joints by a foot or more. Rigid specifications generally require that a certain proportion of the face area of the wall, ranging from one-fifth to one-third, according to the character of the work, shall be formed by headers, and also fix the allowable maximum and minimum dimensions of both headers and stretchers. In figures 7 - 12 are shown various possible arrangements of headers and stretchers for an isolated wall; and in figures 13 - 21 are shown similar arrangements for the case of two walls meeting at an angle, and thus forming a corner or quoin. The plan usually adopted at a corner is to have a stretcher run out alternately into each wall in the successive courses as in figure 13. It is preferable, in order to tie the walls together in the best manner, to avoid joints at the intersection A B (Fig.14) of the inner faces. It is also an advantage that courses shall be continuous through both walls, in order that there may be the same number of bed joints in each, and in consequence a uniform settlement throughout the structure.

In special cases where a stronger bond is desired than that afforded by headers and stretchers alone, as in some light-houses, piers, dams and other works, further expedients are adopted, such as dovetailing, or otherwise interlocking the stones, or using iron dowels, cramps, tie rods, etc. (Fig. 3).

In brick walls the bricks are laid variously with,

(a) Flemish bond, each course made up of alternate headers and stretchers (Fig. 4).
(b) English bond, having alternate courses of headers and stretchers (Fig. 5), and considered stronger than the Flemish bond.
(c) A modification of the English bond in which a course of headers alternates with from four to six courses of stretchers (Fig.6), the arrangement commonly employed in this country.
(d) Other less common arrangements of bricks, such as diagonally across the wall.

8. Classification of Masonry. Stone masonry is commonly classified according to the nicety with which the joint surfaces are prepared, and also according to the methods of arranging the stones in the structure. In all but the most inferior work the stones are imbedded in hydraulic cement mortar, which assists in giving uniform distribution of pressure, unites the whole mass of stone work, and prevents the entrance of water and gases at the joints.

Rubble Work is composed of stones of irregular shape, no especial care being taken to dress the bed or other joint surfaces. The spaces between the larger stones must as a rule be filled in with smaller stones and mortar. If mortar is not used then we have dry rubble. Rubble is variously employed in retaining walls, as backing for the higher classes of masonry, for side walls of sewers, foundations of buildings, etc.

Squared Stone Work is superior to rubble, both in strength and in appearance, and is composed of stones roughly squared, and dressed to give joints one-half inch or more thick. This class of masonry is largely used in heavy retaining walls, abutments, piers, and other massive work. The distinction between it and ashlar is frequently ignored, both being included in the latter class.

Ashlar is a general term applied to cut stone masonry, and in particular to that composed of stones with joint surfaces dressed to give joints less than one-half inch thick. It is employed wherever the highest class of work is desired, as in piers, arches, canal locks, gate chambers, engine foundations, etc., and is often backed with rubble. If the individual stones have all their dimensions assigned in advance of cutting, they are termed dimension stones.

Masonry may further be classified according to the extent to which the stones are arranged in continuous horizontal courses, as:-

(a) Coursed or range work.
(b) Broken range work.
(c) Random work.

In railroad specifications sundry arbitrary classifications of masonry are made, as for example into first, second and third class arch culvert; box culvert, etc.

9. Selection of Stone. The stones most often employed in engineering structures in this country are varieties of granite, sand-stone and lime-stone. Granite occurs in various shades of red and gray and can be given a high polish. It is hard and therefore expensive to cut, but is suited to structures requiring great strength. Its color, hardness, durability, susceptibility to polish, and liability to discoloration, are all largely determined by the structure of the feldspar contained.

Sandstone occurs in gray, brown, red, buff and blue colors, and does not take a polish. It consists of grains of sand, cemented together by either silica, carbonate of lime, or oxide of iron, these materials exerting a controlling influence upon the character of the stone. Silica renders the stone light colored and intensely hard, so that it can be worked only with great difficulty. Carbonate of lime causes it to be soft and crumbly, so that it easily disintegrates. Iron oxide gives a medium and satisfactory hardness, as shown in Portland brown-stone.

Lime-stone, as used for engineering construction, is found in shades of gray and blue, and in general is easily worked. Those varieties sus-

ceptible of high polish are classed as marble, occur in many beautiful colors, and are limited in employment to ornamental construction.

Building stones from different quarries vary widely in their durability under exposure to variable climate and to the gaseous atmosphere of cities, and the behavior, under such conditions, of stone from a new quarry cannot generally be predicted with certainty. Nearly all sandstones are to some extent permeable to moisture, and are hence liable to disintegration if exposed to freezing soon after being quarried; but if allowed to "season" during the warmer months of the year, before exposure to the cold, they withstand freezing weather without difficulty, the quarry water having evaporated. Frequently such stones, and especially limestones, possess the important advantage that when newly quarried they are soft and can be worked into any desired shape with perfect ease, while as they season they grow hard.

10. Shaping the Stone. The rough blocks from which stones are to be cut are to be assumed as approximately rectangular in shape. The workman is furnished with suitable sketches, the necessary dimensions and, if the character of the work requires, with patterns and templets or even models. More or less verbal directions and explanations must also usually be given. It is said that perhaps the most common source of error in difficult stone-cutting is the act of the workman in pushing blindly ahead with his work without fully comprehending the true shape of the stone. Fig. 26 is a copy of a workman's "paper", as given out to him with the rough block, and illustrates the practice of a certain large granite quarry. There are, as a rule, different methods of procedure at command in cutting the stone to shape, especially in difficult work, varying in any particular case as to convenience, the amount of labor required, and the probable accuracy of result.

Commonly the first operation is to dress off some one plane face, which is accomplished by running straight chisel drafts successively along the borders, these drafts being brought to a true plane by applying two straight-edges, of equal width and with parallel sides, to different drafts and sighting across the edges (Fig. 27). The surface enclosed by these preliminary drafts can then be reduced to a plane by the proper tools, the workman testing from time to time by a straight-edge applied in different directions across the surface. The operation here outlined is termed taking out of wind and can next be extended to other plane faces, whether they are at right angles or not, by the aid of a steel square or the bevel, the latter having a movable blade so attached to a straight stock that any desired angle can be set between them.

Any curved surface having rectilinear elements may be obtained in stone-cutting by first chiselling drafts or channels to correspond to two or more directrices, and then cutting the finished surface with reference to these, being guided by a straight-edge applied from time to time to the directrices in the direction of rectilinear elements (Fig. 28). Channels may be cut at short intervals corresponding to such elements, and the intervening surfaces then reduced by eye.

For warped surfaces, so-called twist rules are very commonly employed (Fig. 29). These consist of one or more tapering straight-edges, so framed to a rule having parallel edges that when the upper edges of the rules are in a common plane the lower edges will lie in a warped surface such as that required. Channels are cut along the stone to receive the

straight-edges, and are sunk until, as shown by the above test, the channels have reached the required surface. These channels then serve as directrices, to which straight-edges may be applied for determining elements, as already indicated.

In all this work, especially on stones of irregular shape or involving curved surfaces, the workman is aided in shaping the surfaces and in verifying them by the various patterns and templets. The draftsman who is charged with the duty of preparing these makes a full-size drawing upon a floor or other suitable surface, as previously explained; decides what procedure shall be followed in cutting the stone; cuts such patterns as are needed to the outlines of the faces as shown in the drawing; afterwards trimming them so as to make the proper reduction for joints. They can then be turned over to the stone-cutter, who will work much more closely and uniformly to these patterns than he would to dimensions. Lejeune, whose plates are largely employed in these notes, opposes the use of bevels, as not giving sufficiently accurate results, but on account of their convenience, and the economy of labor which they permit, they are likely to be employed in practice, except perhaps in the more difficult work.

11. Dressing of Stone Surfaces. The selection of a particular style of finish for the face of the stone is partly a matter of taste, and is partly governed by a consideration of the peculiar uses to which the structure is to be put. For the face of a large and massive wall the rock face is appropriate, while for bridge seats, lock walls, gate recesses, etc., where the projections of the rock face would be inadmissible, a smoother surface such as pointed, or patent-hammered, for example, is required. Whatever the style of dressing adopted, even if the face of the stone be left rough, as in rock-faced and quarry-faced work, the edges or arrises are in general worked to definite lines for precision in laying and to give a certain appearance of regularity to the work.

The faces of stones used in rubble masonry are left about as they come from the quarry. The coarser ashlar or squared stone masonry may have the stones either:-

(a) Quarry-faced, the exposed faces being left substantially as given at the quarry.

(b) Pitch-faced, the stone being pitched away from the edges so that these remain clearly defined.

(c) Drafted, a narrow chisel draft being carried along each edge of the face.

Ashlar proper has the edges of the faces of the stones defined either by pitching or by chiseled drafts, while the main surface may be dressed according to any of a great variety of methods, some of the more prominent of which are described below, the style of finish taking its name usually from the tool by which it is directly obtained.

FOR GRANITE SURFACES:-

(a) Rock face, the irregular quarry face of the stone being left, except as modified by pitching. This mode of dressing is now used in most cases where at all admissible.
(b) Pointed, the surface being more or less roughly indented through the use of the point, with hammer or mallet.
(c) Pean-hammered or Pean-axed, the surface being reduced with the pean-hammer or axe, a tool having two opposite cutting edges, and giving a very rough finish, commonly to be seen on granite sidewalk curbing.
(d) Patent-hammered, similar to pean-hammered, but much finer, the finish being obtained by the patent hammer, a double-headed tool containing two sets of chisels, which give at each blow a series of parallel cuts, ranging from 4 to 12 to the inch, according to the fineness desired. The surface is correspondingly known as "4-cut," "5-cut," and so on. The 6-cut finish is the variety of patent-hammered probably most often used in engineering work, and is frequently selected for grooves, waterways, gate-frame seats, etc.

Other tools commonly used in dressing granite are the set or pitching tool, used for dressing edges of a block to line; the spalling hammer, sometimes used for taking off larger projections than can be removed with the set; and various chisels, used for finishing moldings, cutting drafts around rock-faced and pointed work, dressing portions of surfaces not easily accessible with patent hammer, lettering and tracing. The set, point and chisels are driven with the hand-hammer.

The ordinary steps in the process of dressing a granite surface are as follows, these steps being successively pursued as far as necessary to give the desired finish, and the cost, of course, being increased by each operation:-

(1) Dressing edges to line with pitching tool.
(2) Roughing out surface with point.
(3) Cutting down irregularities left by point with pean hammer.
(4) Dressing down with 4, 6, 8, 10 and 12-cut patent hammers, successively, the irregularities left by each preceding tool.

Machinery has been applied to some extent to sawing, polishing, and otherwise dressing granite. Surfaces are prepared for polishing with the 10-cut or 12-cut patent hammer, or in some cases by turning in a special machine, the process of polishing consisting in rubbing first with sand, then with emery, and finally with putty powder.

FOR LIMESTONE SURFACES:-

(a) Rock-face.
(b) Pointed.
(c) Tooled, the surface showing straight, parallel ridges and indentations, made by the chisel.
(d) Drove, a surface somewhat similar to tooled, but showing wavy stripes, and also made with a chisel called the drove.
(e) Rubbed, a smooth finish.

Thickly bedded limestones are commonly sawed into blocks with gang saws. Some thinly bedded stones have beds smooth enough to be used in

ordinary coarse work without dressing, and require only the faces and vertical joints to be dressed.

Marble is usually finished with either a drove, tooled, or polished surface. For carving this and other stones various gouges, chisels, drills and other special tools are employed. The steps in the process of cutting marble are:-

(1) Shaping up the block with spalling hammer and pitching tool.
(2) Roughing out surface with point.
(3) Cutting down projections left by point with <u>tooth-chisel</u>.
(4) Cutting surface smooth with drove.

FOR SANDSTONE SURFACES:

(a) <u>Rock face</u>.
(b) <u>Pointed</u>.
(c) <u>Crandalled</u> and <u>Cross-crandalled</u>, a somewhat regular and fine pointing given by the <u>crandall</u>, a tool having a handle about 2 feet long and keyed into the end a series of double-headed steel points.
(d) <u>Tooled</u>.
(e) <u>Tooth-chiselled</u>.
(f) <u>Bush-hammered</u>, a fine pointed surface given with the <u>bush-hammer</u>, the head of which is a square prism of steel with the end faces cut into pyramidal points.
(g) <u>Rubbed</u>, the stone being at once sawed to a true surface, and then rubbed with another piece of sandstone, sand and water being applied during the operation. This is the finest finish of which sandstone is susceptible, and is often to be seen on the fronts of buildings.

12. <u>Special Constructions</u>. The following constructions not always given in books on drawing may be of service:-

(a) To construct an arc of a circle by points, having given the span and rise, that is to say, having given three points (See Breithof: Coupe des Pierres).
  (1) Given A, B, C, (Fig. 30). Draw chords as shown and upon these lay off the equal lengths A D and C E. At D and E erect perpendiculars on which lay off, above and below the chords, equal spaces. Draw a line from A through the first division above D and one from C through the first division below E. These lines will meet at a point F on the required arc, and other points can similarly be found.
  (2) Given A, B, C, (Fig. 31). From A and C as centers, with equal radii, describe arcs as shown. Sub-divide m s into any number of equal parts, and continue the equal divisions upon the opposite side of the chord A B. Lay off the common arc m a also successively each way from n. Draw a line from A through the first division below m and one from C through the first division above n and these lines will meet at a point F on the required arc, other points being found in a similar manner.

(b) To draw a normal to an ellipse at a given point.

(1) The bisectrix of the angle formed by lines from the foci of the ellipse to the point in question will be a normal (See Fig. 32).

(2) Complete the rectangle O C upon the semi-axes (See Fig. 33) and draw its diagonals. Draw p l perpendicular to A O, meeting O C at l and then draw l n perpendicular to A B meeting A O at n. n p will be a normal.

(c) To find the plane angle which measures the solid angle between any two plane faces:- Assume the solid cut by an auxiliary plane perpendicular to the edge of the solid angle in question. This plane will cut from the adjacent faces lines enclosing the required plane angle. Now conceive a perpendicular in space from the vertex of this plane angle to the H trace of the auxiliary plane. The line will be perpendicular not only to the trace but also to the edge of the solid angle. If the projecting plane of the latter line be now revolved into H the perpendicular will move with it and the true length of the perpendicular will become evident. This having been determined, the revolution of the vertex of the desired plane angle into H is a simple matter and the angle is then revealed in its true size. Thus in Fig. 34, where the various planes are given by their traces, draw t - t, the H trace of the auxiliary cutting plane; revolve m into H as shown, and the line a b, perpendicular to the revolved position of m, will be the true distance in space from a point in the horizontal plane at a to the edge of the solid angle. c d e then represents the required angle revolved into H, and therefore shown in its true size. The procedure would be similar for working in V.

13. Walls with Plane Sloping Surfaces. A wall having a sloping face is said to have a batter, and this is usually expressed in inches per foot, a batter of 1 inch to the foot, or 1 in 12, referring to a slope at the rate of 1 horizontal to 12 vertical. Such a wall is shown in Fig. 22, where the batter is the ratio of the offset F M to the corresponding height M B. A stone of the lower course in this wall may be cut as follows:- Let a rectangular block be assumed, of width at least equal to A B, of height A E, and of any suitable length. Let A B M E E'A'B'M' (Fig. 23) be such a block. First dress the lower bed A B B'A'; then, working from this surface with the square, dress the vertical face A E E'A', and the joints A B M E and A'B'M'E', which should be square not only with the lower bed but with the vertical end face. Next dress the upper bed E M M'E' square with the vertical faces and at the proper height above the lower bed. It remains only to cut the battered face, which may be done by first laying off A B and A'B' equal to A B (Fig. 22), erecting perpendiculars at B and B', then making M F and M'F' equal to M F (Fig. 22), drawing the lines B B', B F, B'F', F F', and finally cutting away the triangular prism B B'M M'F F'.

A preferable mode of working is as follows:- dress a vertical joint surface first, tracing upon it by means of a pattern the outline A B F E (Fig. 22). Then, working from this surface with the square, dress the end faces and the bed joints, finally cutting the other vertical joints parallel to the one first dressed.

If the batter is considerable, and it is desired to avoid acute angles between the bed joints and the battered face, this may be done by stopping the horizontal beds a few inches distant from the sloping face, and then carrying them out at right angles to that surface, as shown at G D H and E C F (Fig. 24). The bottom stone could be finished as shown by the profile F N M B.

A stone of the bottom course can in this case be cut as follows:- Choose a block having the height B I, the width A B (Fig. 24), and any suitable length. Let a b q p a'b'q'p' (Fig. 25) be such a block. Begin by dressing the face which is to form the lower bed, and upon this trace the rectangle a b b'a', having the sides a b and a'b' equal to A B (Fig. 24) and the sides a a' and b b' equal to the assumed length. Next, by means of the square dress a plane face a b q p perpendicular to the lower bed and passing through the edge a b. Upon this face, by means of a pattern, trace the contour a e c f n m b, identical with A E C F N M B of Fig. 24. Then, by aid of the square, cut all the faces that are perpendicular to a e c f n m b, giving to these faces the assumed length, which will permit of easily cutting the remaining face a'e'c'f'n'm'b'.

If it is desired to work more accurately, one may cut not only a b q p but a'b'q'p' square with the lower bed, tracing upon a'b'q'p' the contour a'e'c'f'n'm'b' identical with a e c f n m b. Thus for each of the remaining faces the workman will have two directrices in the same plane, such as a e and a'e'; e c and e'c'; c f and c'f', etc., and the faces can be cut with ease and precision.

It should be noticed that the broken joint E C F produces a salient angle in the second course and a re-entrant angle in the first course. It is somewhat difficult to cut the latter angle with accuracy, and it is also likely that the two angles will not be cut precisely alike, in which case there will be imperfect contact between the faces and a liability to fracture during settlement. Special care should, therefore, be taken with the cutting of broken joints.

14. Walls with Cylindrical Surfaces. The first case to be considered will be that of a wall whose two faces are circular cylinders with concentric bases.

Let A B C D and E G F H (Fig. 35) be concentric circles with a common centre at O. The horizontal projection of the wall will be in the annular space between the two circumferences. The height of the wall being assumed, the vertical projection is easily obtained as shown. Now divide the circumference A C B D into a certain number of equal parts, 8 for example, and through the points of division draw the lines A E, M N, C G, etc., all converging toward O and forming the horizontal projection of the vertical joints of alternate courses, beginning with the upper. Then divide anew A C B D into 8 equal parts in such a way that the points of division shall fall midway between those first obtained, and through the new points of division also draw right lines converging toward O. These lines will be the horizontal projections of the vertical joints of courses alternating with those first mentioned. Finally, draw horizontals such as P Q at suitable intervals to represent the bed joints of the courses, and through the points of division of A C B D draw perpendiculars to the ground line to determine the vertical joints in the elevation.

A stone such as that having for horizontal projection the figure M N G C and whose exterior face is vertically projected in M'M"C'C"

could be cut as follows:- Cut a pattern to the contour M N G C, and choose a block of stone having the height M'M", and its other dimensions such that the pattern can be applied to either of the quarry beds p t s c' (Fig. 36). By means of the pattern trace upon one quarry bed (assumed to have been brought to a plane) the contour m'n'g'c'. Then through the points m',n',g', draw upon the corresponding faces of the stone the right lines m'm, n'n, g'g, perpendicular to the quarry bed, apply the pattern to the face q u r c so that the vertices shall fall at the points n, m, g, c, and trace the outline of the pattern.

The vertical joints, which are plane faces, are easily cut, since for each of them we have four right lines upon which the straight-edge may be applied. The cylindrical surfaces m'c'c m and n'g'g n can be cut by working so that the straight-edge always rests upon the two circular arcs limiting the faces at top and bottom, maintaining the straight-edge parallel to itself in the successive positions. This can be accomplished by dividing the arcs m c, m'c', for example, into the same number of equal parts c'x = c y, x v = y z, v m'= z m, and applying the straight-edge so that it shall occupy the positions x y, v z, etc. In other words, the straight-edge should always be directed along one of the elements of the cylinder.

The mode of cutting above explained is disadvantageous in occasioning a considerable loss of stone, and is therefore inferior to the following:- Choose a block having the height of the course, and its other dimensions such that the pattern can be applied in the position m'n'g'c' (Fig. 37), trace the right lines m'm, n'n, g'g, c'c, perpendicular to the quarry beds, and finish the stone as before with straight-edge and square.

It is plain that a block of smaller dimensions than before used answers in this second method. The difference is shown in Fig. 38, where p t s c' represents the size of rectangle required in the first case, and t p's'c" that in the second case, evidently smaller than the other.

15. Cylindrical Wall with Elliptical Base. If it is desired that the interior face of the wall be a cylinder having for its base an ellipse similar to that which determines the exterior face, an ellipse will be drawn having its axes proportional to those of the exterior ellipse. The wall will in this case evidently be thicker in the direction of the major axes than in that of the minor axes. The vertical joints can no longer, as in the case of a cylindrical wall with circular base, converge towards the common centre of the two faces, since they would then be perpendicular to neither of the ellipses, except in the line of the axes. Further, if the joints were made normal to one ellipse, they would not be normal to the other. This difficulty may be avoided by using broken joints, one part being normal to the exterior curve, and the other normal to the interior. Broken joints have been stated (13) to be objectionable where pressure is brought to bear upon them; but for vertical joints, as in the present case, with simply contact, there is no disadvantage in them. If, however, the wall were in any way exposed to a thrust, bringing pressure upon the broken joints, it would be better to substitute for them plane joints normal to a mean ellipse situated between the other two. In this case the joints would be nearly normal to each surface.

If it is desired that the interior face of the wall have for a base a curve everywhere equidistant from the exterior ellipse, construct a suitable number of normals to that ellipse, upon each of which lay off a distance equal to the desired thickness of wall.

The cutting of the stones for an elliptical wall will be prosecuted in precisely the same manner as for a circular wall, employing one or the other of the methods previously explained (14).

16. Walls with Conical Surfaces. If it were required to join two walls having the same batter, in such a way as to avoid an angle at the junction, it would be convenient to employ a conical surface in the following manner:- Let Fig. 39 show the plan of two walls of the same batter, with the right section of one of them. Join the exterior faces of these walls by a conical surface, of which the vertex will be horizontally projected at S, at the point where the two right lines A B and C D meet, and vertically projected at the point of intersection of A'E' and C'F', a point lying, in this case, outside the paper.

The conical surface will evidently be that of a right cone with a circular base, which will be cut by the planes of the bed joints of the courses along portions of horizontal circles, of which the centers, situated upon the axis of the cone, will all be horizontally projected at the same point S, which is, therefore, the horizontal projection of the vertex of the cone. All these circles will be concentric in horizontal projection.

The vertical joints will be indicated as shown in the drawing, taking care to make them alternate in the successive courses. If it is determined entirely to avoid the acute angles which the bed joints of the stone make with the exterior face of the wall, then one will make the joints, for a short distance from the sloping surface, perpendicular to that surface, thus producing conical surfaces all having their vertices upon the axis of the cone which forms the exterior face of the wall. It is rare, however, that the batter of the wall is sufficient to make this construction advisable. It is clear that the interior face of the conical wall will be a right circular cylinder having for a radius S B, and tangent to the faces of the two walls.

If it is proposed to cut the stone whose horizontal projection is limited by the figure G H N M (Fig. 39), choose a block of stone having a height between quarry beds equal to the height M'M" of the course. Having dressed the lower face of this block to a plane, trace upon it, by means of a pattern cut along G H N M, the contour g h n m (Fig. 40); through the points m and n draw perpendiculars to the bed, and to the upper bed apply a second pattern cut along I M N K, thus tracing the contour i m'n'k. Finally, draw i g and k h. The interior face, which is cylindrical, will be cut as has been directed for cylindrical walls (14); that is to say, the workman will be guided by a straight-edge applied to the arcs m n, m'n' in such a manner that in each position it is parallel to m m' and n n'. The exterior face, which is conical, will be cut by using, to verify the work, a straight-edge which will be made to pass through corresponding points of division of the arcs i k and g h, which preferably will have been divided into the same number of equal parts. The joint surfaces, which are planes, will be easily dressed.

Just as was explained for cylindrical walls, there is here a second method of cutting, more economical in the use of stone. Fig. 41 will make clear the features of the second method.

17. Oblique Conical Wall. When two straight walls of different batter are to be joined by a curved wall, the surface of the latter naturally is made that of an oblique cone with a circular base. The same surface is also sometimes employed for the end of a bridge pier, to join the slightly battered side surfaces with the line of the cut-water.

Let A B and A C (Fig. 42) be the horizontal traces, prolonged, of the battered faces of two walls which are to be joined and whose right sections are shown; D E and D F the horizontal projections of the upper edges of those faces, and G H and G I the horizontal traces of the vertical faces of the walls. Through A draw a right line A o dividing the angle C A B into two equal parts. Every point on A o will be equally distant from the sides of the angle; consequently, from points upon A o circles may be drawn tangent to both sides of the angle. Take then a point o, distant from the sides A B and A C by an amount equal to the radius desired for the base of the cone, and the arc M N, having o for a center, will be tangent to A B and A C.

Similarly, bisect the angle E D F, and upon the line D o_4 thus obtained take a point o_4 distant from the sides D E and D F by an amount equal to the radius desired for the arc which is to join the upper edges of the sloping faces. From o_4 as a center describe the arc P Q, and draw the right lines M P and N Q. These lines meet at a point S which is the horizontal projection of the apex of the oblique cone which is to join the two sloping faces and which will be tangent to them. If now we draw a right line through o and o_4 it will pass through S, since in an oblique cone with a circular base all plane sections parallel to the base are circles having their centres upon a straight line passing through the apex. o S is the horizontal projection of this line.

It is evident that the circles which are to connect the joints separating the courses will have their centres upon o S, between o and o_4, at intervals proportional to the heights of the courses. The vertical joints of the conical portion of the wall could be determined by vertical planes perpendicular to the trace of the cone; but it is preferable to make them perpendicular to a horizontal section at mid-height of each course.

In order to cut one of the stones of the conical wall, that for example horizontally projected in m n q t r p (Fig. 42) we may proceed as follows:- Dress the two bed joints and with a pattern trace upon the lower the contour m n t r (Fig. 43) and continue as explained in No. 16. In dressing the conical face mark upon the stone the points where the edges p q, q n, n m, and m p are crossed by a suitable number of elements of the cone, working accurately along the lines thus determined and dressing intervening portions of the surface by eye.

18. The Flat Arch or Plate-band. This is a term applied to an arrangement of wedge-shaped stones having a plane soffit and spanning a door, window or other opening. The sides of the opening are called the jambs. The division of the arch into separate stones is effected as follows:- Upon the right line A B (Fig. 45) which represents the vertical trace of the plane forming the top of the opening construct an equilateral triangle O A B; then through the vertex O draw the lines O F E, O H G, O K I, etc., cutting the line A B at points F, H, K, etc. which divide that line into an odd number of equal parts A F = F H = H K, etc.,

the number of divisions being decided by judgment.

Experience has shown that when there is a tendency for the flat arch to fail under the load which it has to support it is by a rotation resulting in the opening, underneath, of the joints on either side of the key; and, above, of the joints near the jambs, as illustrated in Fig. 46. To avoid such results various expedients are adopted, such as increasing the thickness at the key, diverting pressure by means of a relieving arch built above the flat arch, tying together the stones of the latter by an iron rod, etc.

The slipping of the extreme stones is sometimes prevented by breaking the outer joint along a horizontal plane a b (Fig. 45). The same arrangement may be continued toward the centre, taking care not to give too great length to the horizontal part of the joint, so that it may better resist unequal pressure resulting from settlement. It is thought best, however, to leave the key stone with unbroken joints, that it may settle firmly into place, with perfect contact on either side.

A somewhat different arrangement is sometimes employed in which the joints are broken as along the line f g h n (Fig. 45), each stone catching upon the adjacent one by a short offset g h n. As the appearance is not entirely pleasing to the eye, the offset may be made for only a part of the thickness of the flat arch, thus giving a projection and indent as shown in figures 47 and 48.

In order to avoid the disadvantage of giving the stones acute angles at the intrados, vertical cuts such as x y may be made at the points of division of A B (Fig. 45), meeting a horizontal line p q distant a short way from A B, the joints being continued beyond from the direction of O.

In Fig. 49 let u v and x y be the horizontal traces of the faces of a right wall to be pierced by a flat arch; A A" C C" E the horizontal trace or outline proposed for the side of the opening; A'B' the vertical projection of one-half the intrados, and A'P' that of the face of the jamb. Draw C'D' and C'K' respectively parallel to A'B' and A'P', and distant from them by an amount equal to C A". Above E erect the vertical V'E', prolonging it to a point E" situated above C'D' at a distance arbitrarily chosen, and through E" draw a parallel to A'B'. Then divide the entire line A'B' representing the intrados into a suitable odd number of equal parts, and through the points of division erect the verticals A'L', M'G , etc., afterward drawing the lines L'T', G'Q', etc., all radiating from O, a point determined as previously explained. Through T' draw a horizontal T'S', and by projection determine in plan the lines A L J, M G H, etc.

An arch stone, such as that vertically projected in R'Q'G'M'A'L'T'S' (Fig. 49) may be cut as follows:- Starting with a stone of a length equal at least to the thickness v y of the wall, and with the other two dimensions sufficient to contain the pattern of the head R'Q'G'M'A'L'T'S', dress the joint surface u y y'n' (Fig. 50); then squaring from this face cut the two faces u v x y and u'v'x'y' (Fig. 50). Now, upon one of these faces apply a pattern of the head and trace the contour r q d c f b t s, after which draw through the points r, q, c, s, which are upon the edges of the face u v x y, right lines r r', q q', c m', s s', which meet the edges of the other face at points r', q', m', s', which serve as guides for applying a pattern and tracing the contour r'q'g'm'a'l't's'. The stone will now be cut as though it were to be a prism having for bases

the contours above determined by the patterns. This prism having been cut, lay off the distances a'a and m'm, draw a m, and through a and m draw, square with the edges of the intrados, the lines a l' and m g'; then lay off l l' and g g' and join l with g; next draw h j parallel to c f at a distance from it equal to B' z (Fig. 49); and finally join j with l and h with g. It remains then simply to cut the three faces h j l g, g l l'g', and g'l'a m which form the embrasure.

In order to cut a skewback stone, begin as though there were no embrasure, thus obtaining a prism of which the bases u x a l t and u'x'a' l't' are determined by means of a pattern cut along the contour U'X'A'L'T' (Fig. 49). That accomplished, trace upon the bed joint of the stone the contour x e'y'y a"a'x' by means of a pattern cut to the horizontal projection of the skewback, that is to say, to the contour X E C"C A"A U (Fig. 49). Through e' (Fig. 51) draw the right line e'e square with the edge a x; make e e' equal to E'E" (Fig. 49); through e (Fig. 51) draw e j parallel to a x; through a" draw a"n square with a'a"; lay off n m, and through j and m draw j m. The skewback is then entirely outlined and the cutting is easily accomplished.

If the flat arch is in a wall with battered face, begin by cutting the stone as though it were in a right wall, and then give the face the proper batter as indicated in connection with Fig. 23. If the flat arch is in a cylindrical or conical wall, again proceed as though it were in a right wall, afterward cutting to the proper curvature as has been explained in Secs. 14, 16 and 17.

The Flat Vault (French, Voûte Plate) is a vault having for its intrados a plane surface, and therefore is somewhat analagous to the flat arch. This construction will not here be considered in detail, but is illustrated in Figs. 52 - 57.

19. Wing Walls:- The abutment of a dam or bridge is usually flanked by a wing wall, serving in part as a retaining wall to support the embankment behind it and in part as a protection of the latter from scour. The height of the wing wall usually decreases toward the end, in the case of a bridge abutment at least, either in successive steps or by a uniform slope given to its top, and the thickness at the base diminishes accordingly. The stones forming the steps of the slope are continued well under those of the overlying course, and when the slope is covered with a coping it is often finished at the foot by a heavy newel stone. Wing walls take a great variety of shapes in plan and meet the abutment at various angles, depending upon the conditions peculiar to each case. Preceding plates will serve to illustrate this construction, which is arbitrary as to various details.

20. Pyramidal Buttress:- It is sometimes desired to reinforce a wall by buttresses projecting from its face. Let it be required to make the drawings for a buttress of which the transverse section, the outline of the base, and the batter of the faces are assumed to be given.

The right section of the buttress and of the wall against which it rests, and the outline R M K V (Fig. 64) of the base of the buttress, can be drawn at once from the given dimensions. It remains to find the intersections of the various faces. To determine the direction of M N,

imagine the adjacent faces of the buttress, produced if necessary, cut at any convenient height by an auxiliary plane along a b and c d. These lines are respectively parallel to R M and M K at distances e f and g h determined by the height of the auxiliary plane and the batter of the faces. M N passes through the intersection of a b and c d. The direction of R S is similarly found from the intersection of a b and t u. The positions of N O and S P with reference to M K are given by the right section, and the main lines of the drawing are then fully determined.

The manner of drawing the joint lines is easily seen. The line P V is divided proportionally to the height of the courses, and through the points of division the horizontal joint lines are drawn parallel to the outline of the base. The vertical joint lines show in plan at right angles to the horizontal joints, In elevation they appear inclined on the side face of the buttress and should be made parallel to some well determined guide line, such as P'y'.

The uppermost stone of the buttress is shown in isometric view (Fig. 65), and can be cut from a rough block as follows:- First dress the lower bed A_1 B_1 C_1 D_1 and trace its outline by the pattern. Next cut the front and rear faces by using the bevel from the base already cut, and by patterns trace the outlines A_1 D_1 P_1 S_1 and B_1 C_1 O_1 N_1. Two lines of the upper face are now determined and the face can be cut and, by applying a pattern, its entire outline can be traced. Finally dress off the lateral faces A_1 B_1 N_1 S_1 and D_1 C_1 O_1 P_1, the bounding edges of which have all been determined by the other faces.

MASONRY ARCHES.

21. An arch is a curved structure having a cylindrical intrados, and ordinarily forming the roof over an opening. If the arch is of stone it is composed of wedge-shaped blocks, which are sustained in position by their mutual pressure. If the length of the opening in the direction of the axis is considerable, as when it is more than a mere opening through a wall, the arch is in architecture classified as a vault, although this distinction is not commonly observed in engineering.

Arches are classified according to various features, and especially as to the shape of a right section of the intrados, in this way giving rise to the terms semi-circular arch, elliptical, etc.

The semi-circular arch, also called Round and Full-centered, has its intrados a semi-circle in right section. A right section of the Segmental arch shows a segment less than a semi-circle. These are the most common forms, the semi-circular being considered the more perfect type as to appearance, but weaker than the segmental and requiring greater vertical space, according to the length of span. The segmental arch exerts greater thrust than the semi-circular upon pier or abutment, but is itself strong, may be proportioned to occupy but moderate vertical space, and is the form usually selected for large spans.

If the rise of the arch is less than the half-span, as in the case of an elliptical arch with minor axis vertical, the arch is sometimes described as surbased; if the reverse is true, as in the case of an elliptical arch with major axis vertical, the arch is said to be surmounted. If the elements of the intrados are horizontal, the case is that of a

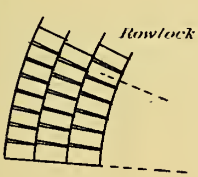
horizontal arch; but if they are inclined to the horizontal plane the arch is termed a *descent*.

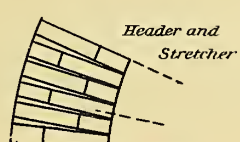
As has been said, the voussoirs of an arch are wedge-shaped, the joints radiating from the centre or centres of curvature of the intrados, this direction making them also normal, or approximately so, to the line of pressure in the arch. Although intrados and extrados are often designed as concentric surfaces, especially for small spans, they are by no means always so, for the conditions of stability may require an increased thickness of the arch toward the springing line, and in any event the extrados is likely to be left rough in construction, except in the case of the ring stones.

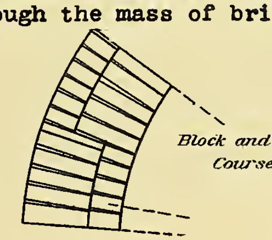
In French practice the extrados is often arranged as follows: After having fixed the thickness B b (Fig. 63) to be given at the crown, which thickness varies with the span, the weight to be supported and the material of the voussoirs, and is governed by recognized rules and precedents, take upon the vertical B O a length B O' equal to from two-thirds to three-fourths the span A C; then with O' b as a radius describe the arc a b c, which defines the extrados.

The precise shape of the voussoirs, as well as the precise number into which the arch shall be divided, are largely matters of taste. In general the voussoirs should be somewhat deeper (radially) than they are wide, and in order to insure a key stone at the centre, there must be an odd number of them. Their width as measured on the intrados is sometimes increased moderately from the key toward the springing lines. Sometimes the joints of the arch stones and the coursing of the adjacent exterior masonry are planned with reference to each other, and sometimes independently. Most often each voussoir is terminated by a horizontal face P Q (Fig. 62) and a vertical face Q R, the joints of the voussoirs being adjusted in some degree to the coursing joints of the wall in which the arch is built. Sometimes the joints are arranged as in the left-hand part of Fig. 62, but this is objectionable. The various patterns and templets required in cutting the voussoirs are to be made from full-size drawings, but often it is also advisable, especially for large and important structures, to compute the dimensions.

Arches of brick are of very common occurrence, being employed in aqueducts, sewers, buildings, and even sometimes for bridges of considerable spans. In engineering work the bricks are as a rule arranged in independent concentric rings, joined to one another by mortar only, and giving the so-called *Rowlock* bond. In ornamental construction, as in the fronts of buildings, the bricks are frequently laid in the *Header* and *Stretcher* bond, the joints being continuous from intrados to extrados. Such an arrangement involves either "rubbing" the bricks to a wedge-shape, or permitting wide joints at the extrados. The *Block and Course* bond and other devices have been employed to some extent in engineering structures with a view to breaking the continuous concentric joints of the rowlock bond, and by grouping the bricks in the shape of stone voussoirs to cause the pressure to be more regularly distributed through the mass of brickwork.

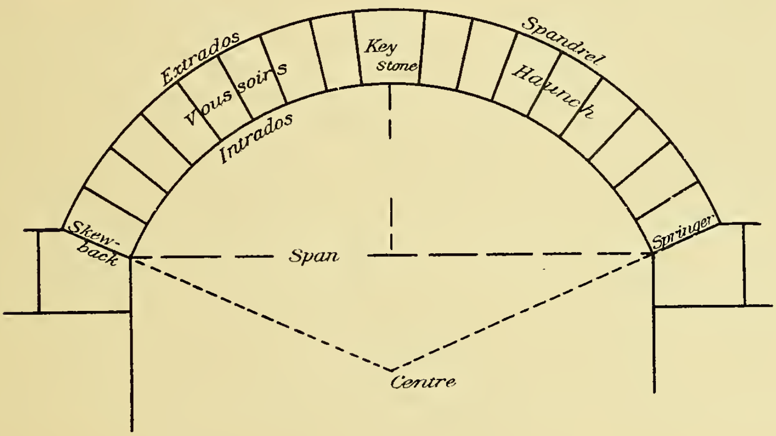






The following are common technical terms applied to the various parts of the arch:-

Intrados or Soffit. The inner surface of the arch.
Extrados or Back. The outer surface of the arch.
Crown. The highest part of the arch.
Springing Line. The line in which the intrados meets pier or abutment.
Span. The perpendicular distance between springing lines.
Rise. The height of intrados at crown above level of springing lines.
Voussoirs or Arch stones. The stones composing the arch.
Ring Stones. Stones showing in the face of the arch.
Key Stone. The middle ring stone.
Arch Sheeting. The material forming the lining of the arch, especially that portion not showing in the faces.
Springer. The voussoir nearest the springing line.
Skewback. The inclined surface of the stone from which a segmental arch springs. The term is also sometimes applied to the stone itself.
Haunch. The portion of the arch midway between crown and springing line.
Spandrel. The lateral space outside the extrados.
Course. A row of voussoirs running in the direction of the axis of the arch.
Coursing Joint. A joint separating adjacent courses.
Heading Joint. A joint running transverse to the courses.



22. Right Arch. This term is applied to a horizontal arch whose faces are perpendicular to the axis. In Fig. 60 is shown the elevation of such an arch, with voussoirs arranged in accordance with principles already stated.

In order to cut the voussoir projected in M N P·D C one may proceed as follows:- Choose a block of stone having a length at least equal to

the thickness of the wall (if the opening is not too long to be covered by a single length of stone; otherwise choose any suitable length, depending upon circumstances), and whose base x u d t (Fig. 61) can contain a pattern of the head M C D P N (Fig. 60). Dress the face x u d t and by means of a pattern of the face of the voussoir trace the contour m c d p n, preferably arranging that the edge of one of the joints shall coincide with the quarry bed u v d'd. Now cut the joint surface d p p'd' passing through the right line d p square with the face of the voussoir, and outline the contour of the joint surface D P by constructing the right angles p'p d and p d d', of which the edges p p' and d d' are equal to the length of the stone. Proceed similarly for the joint surface M C. Then dress the second face of the voussoir, m'c'd'p'n', which will be easy, since two right lines p'd' and m'c' contained in its plane are known, and the work can be verified by squaring from the joint surfaces already cut. The exact outline will be obtained by applying the pattern M C D P N (Fig. 60) so that M, C, D and P shall fall at their appropriate positions. The faces n p p'n' and n m m'n' can now easily be cut, since for each of them three right lines are known, to which the straight-edge can be applied; they should further be verified with the square, applied to the end faces of the voussoir. The cylindrical intrados can be cut by applying a straight-edge to corresponding reference points spaced at equal intervals on the end curves.

23. <u>A Horizontal Arch, Terminated at one end by a Sloping Skew Face, and at the other by another Arch of Larger Radius</u>:-

The axes of the arches are supposed to be at right angles to each other, and their springing lines to lie in the same plane, which will be taken as the horizontal plane of projection. Both arches are here assumed to be semi-circular, and the larger to be constructed of brick masonry, not therefore requiring an adjustment to its courses of the voussoirs of the other arch, as would be the case if both were of stone. It is plain that the smaller arch will meet the larger along a curve of double curvature, and the sloping face along an ellipse, and it is between these curves that the various voussoirs to be planned will be comprised.

The vertical plane of projection will be taken perpendicular to the axis of the smaller arch. Upon this plane trace the semi-circle F'G'E' (Fig. 66) a right section of the intrados of the smaller arch, and upon the horizontal plane the right lines D F E C, D"F"E"C" which will be, respectively, the trace of the sloping face and a springing line of the larger arch. Divide F'G'E' into an odd number of equal parts, five for example, and through the points of division and the axis O' O" pass planes M'O'O", N'O'O", etc., which will form the joints of the voussoirs and will cut the cylindrical intrados along elements. Limit the joints M'R', N'P', etc., by a circle D'R'C' and terminate each voussoir by a horizontal and a vertical face such as R'Q' and Q'P'.

One of the voussoirs will be vertically projected in the polygon M'N'P'Q'R' and will occupy in the right prism which has this polygon for a base the space between the slope F E and the intrados of the larger arch springing from F"E". It remains to find two things: 1st, the intersection of the faces of the prism with the sloping face; and 2d, the intersection of the same faces with the intrados of the larger arch.

In order to find the first of these intersections, imagine the sloping face cut at D by a vertical plane perpendicular to D E; it is thus

intersected along a line horizontally projected in D A and which will make with a vertical at D an angle which measures the inclination of the face. Let Z D'A' represent this angle, here assigned in advance, turned and projected upon the vertical plane of projection. We may now find the horizontal projection of any point lying on the sloping face, the point vertically projected in M' for example, by conceiving a horizontal line passing through the point and lying in the sloping face, meeting the line D'A' at a point m' horizontally projected upon D A" at m. This point is then to be revolved into the plane D A by means of the arc m m", and then through the point m", thus obtained, draw m"M parallel to D C, meeting the vertical dropped from M' at a point M, which will be the required horizontal projection. The other necessary points may be found in the same manner, after which draw the right lines M R, R Q, Q P, P N, forming the polygon M N P Q R, which will be the horizontal projection of one end of the voussoir. It is to be noticed that the right line R Q ought to be parallel to D C, since the face which has R Q for an edge is horizontal; also that the face which is vertically projected in P'Q', being perpendicular to the vertical plane, will have all its edges horizontally projected upon the same right line perpendicular to the ground line.

The other face of the voussoir, that formed upon the intrados of the large arch, will be obtained in a manner similar to the above. That is, draw in the vertical plane an arc of a circle C'Y corresponding to the curvature of the larger arch; then through the different points such as N', Q', etc., draw the horizontals N'n', Q'q', meeting C'Y at points n', q', horizontally projected at n and q; then by means of arcs revolve these points into the vertical plane C C" and draw through the revolved points parallels to D"C", thus obtaining the desired horizontal projections N and Q. Other points may be found in the same manner.

The joint surfaces meet the intrados of the large arch along curves which are portions of ellipses. The edges of the joints furnish certain points upon these curves, but others may be obtained, if needed, directly by projection, as illustrated for the point X. Since all the joint planes pass through the axis O O" of the smaller arch, it is plain that all the right lines such as M R and N P should converge to the point O; the elliptical arcs M"R", N"P", etc., should for the same reason pass through O", where they ought, further, to be tangent to D"C".

24. Obtaining the Patterns:- The arch has now been determined, since the projections of the faces of the voussoirs have been found. But, for the purpose of cutting the stones, it is further necessary to know each of the faces in its true size, at least the intradosal and joint faces. Now all the intradosal faces are parts of the same cylindrical surface, and if this be developed they will all be known in their true size. To this end, first rectify the curve of right section by laying off upon the indefinite right line e f (Fig. 67) distances e n, n m, etc., equal to the lengths of the arcs E'N', N'M' (Fig. 66). This may be done by laying off with the spacers a short chord the proper number of times, or more accurately by laying off the computed length of the semi-circumference and dividing it into the proper number of equal parts. At the various points of division lay off indefinite perpendiculars from e f, upon these take e e'= I E, n n'= H N, m m'= K M, ... f f'= L F, and through the points e', n', m', ... f', thus obtained, draw the curve e'n'm'f', which will be the development of the arc projected in F M N E. Points upon this curve intermediate between those above mentioned may be found by taking

elements of the cylindrical intrados lying between the edges of the voussoirs, locating these on the development and laying off the proper distances upon them, just as was done for n', m', etc.

In a manner similar to the above lay off the distances e e"= I'E", n n"= H N", m m"= K M" .... f f"= L F", and through the points e", n", m" .... f" thus obtained pass a curve e"n"m"f", which will be the development of the line of intersection of the two arches, horizontally projected in E"N"M"F" (Fig. 66). The true form and size of all the intradosal surfaces are now known, and are represented by the curvilinear figures e'n'n"e", n'm'm"n", etc.

In order to obtain patterns of the plane joint surfaces, lay off upon e f distances n p = N'P', m r = M'R', etc., erect through the points p, r, etc., thus obtained, perpendiculars, upon which take p p'= W P, p p"= W P", r r'= T R, r r"= T R", etc., and the right lines n'p', m'r', etc., will be the outer edges of the joints. To obtain the opposite curved edges, find for each of them at least one point between n" and p", m" and r", etc., which may be done by applying the above method to horizontal right lines passing through the middle of the joints N'P', M'R', etc. (Fig. 66).

25. <u>Cutting the Voussoirs</u>:- For cutting the voussoirs two distinct methods are applicable, known respectively as the <u>method by squaring</u> and the <u>method by bevels</u>. The method by squaring has the advantage of offering the greater precision, but both methods will be explained by supposing them applied to the voussoir vertically projected in M'N'P'Q'R' (Fig. 66).

<u>Method by Squaring</u>:- Choose a block of stone having a length at least equal to the greatest dimension Q"v (Fig. 66) of the horizontal projection of the voussoir, and with its other dimensions such that a pattern of the face M'N'P'Q'R' can be applied upon the two end faces.

Begin by dressing one of the ends to a true plane surface, and trace upon it the contour m_1 N_1 p_1 q_1 r_1 (Fig. 68) by means of a pattern cut to the vertical projection M'N'P'Q'R' (Fig. 66), preferably turning this pattern so that the side M'R' of one of the joints shall coincide with the quarry bed m_1 r_1 R_2 M_2. Then, by means of the square, cut a plane surface from m_1 r_1 exactly perpendicular to the end already cut, and apply the pattern m'r'r"m" (Fig. 67) of the upper joint, placing it at distances m_1 M_1 = M u, r_1 R_1 = R z (Fig. 66), u N being parallel to the ground line, and thus outline the contour R_1 M_1 M_2 R_2 of the joint. Proceed similarly for the joint surface whose edge is N_1 P_1, applying the corresponding pattern n'p'p"n" (Fig. 67) and tracing the contour P_1 N_1 N_2 P_2 of the lower joint.

The intrados is a portion of a cylinder passing through the curve m_1 N_1 and having its elements perpendicular to the base m_1 N_1 p_1 q_1 r_1. The square may answer for cutting this cylinder; but if too short for the purpose a templet may be employed, cut to the arc M'N' and moved upon the two right lines m_1 M_2, n_1 N_2, already traced, in such a manner as always to be parallel to the plane of the arc m_1 N_1. When the intrados has been cut, apply its pattern m'n'n"m" (Fig. 67) which should be of flexible material in order to allow of close contact with the concave surface, and trace the end curves M_1 N_1 and M_2 N_2.

The lateral face q_1 p_1 P_2 Q_2 and the upper face q_1 r_1 R_2 Q_2, being planes perpendicular to the base of the prism, and passing through two known right lines, may easily be cut by the aid of a straight-edge. This done, lay off the distances q_1 Q_1 and q_1 Q_2 respectively equal to v Q and

v Q" (Fig. 66), which will permit of tracing the right lines R_1 Q_1 and Q_1 P_1, and also the arc Q_2 P_2 by means of a templet cut to the curvature of the right section C'Y (Fig. 66).

It remains only to cut that face of the voussoir which lies in the intrados of the larger arch, and of which we already know the contour M_2 N_2 P_2 Q_2 R_2. This face is a cylindrical surface and can therefore be cut by moving a straight-edge upon the contour M_2 N_2 P_2 Q_2 R_2 in such a manner that it passes at the same time through marks a and b, a' and b', M_2 and b"', etc., corresponding to elements of the cylinder. These marks can easily be determined by tracing upon the base m_1 n_1 P_1 Q_1 R_1 right lines parallel to q_1 r_1, such as a_1 b_1, and bringing forward the points a_1 and b_1 to a' and b' by means of right lines a_1 a' and b_1 b' parallel to the edges of the voussoir. The opposite end of the voussoir, being entirely a plane surface, with its exact contour known, can easily be cut by the aid of a straight-edge, cutting away the excess of stone between the face and the base m_1 N_1 p_1 q_1 r_1.

Modified Method:- The method above described can be modified as follows, with the attainment, perhaps, of greater accuracy, and with avoidance of the necessity for using a variety of patterns:- Record upon a rough sketch the lengths of the different right lines M u, M"u, R z, R"z, P v, P"v, etc., and of any intermediate ones desired. Then having squared a right prism having for its base the contour m_1 N_1 p_1 q_1 r_1 (Fig. 68), mark upon the stone the distances r_1 R_1, r_1 R_2, m_1 M_1, m_1 M_2 etc., as recorded upon the sketch; also trace the curves M_2 R_2, M_2 N_2, etc., of which one or two intermediate points should be known.

Method by Bevels:- After having dressed a quarry bed and having traced upon it the contour M_1 M_2 R_2 R_1 (Fig. 68) of the upper bed joint, cut an indefinite plane surface passing through M_1 M_2 and making with the joint already cut an angle equal to R'M'N' (Fig. 66). This will serve provisionally as a substitute for the true intrados, and the proper angle will be obtained in cutting by the use of the bevel, held always in a plane at right angles to the edge M_1 M_2. Having dressed the indefinite plane, trace upon it the rectilinear contour M_1 M_2 N_2 N_1 easily determined from the drawing. The plane would cut the intrados of the large arch along a curve, and not along the right line M_2 N_2, but that is of no consequence, since the plane is to serve but a temporary purpose.

In the same way, by means of a bevel forming an angle equal to M'N'P' (in this case equal also to N'M'R'), one of its sides being kept upon the flat intrados and its plane being always maintained perpendicular to the edge N_1 N_2, cut the lower bed joint, upon which trace the contour N_1 N_2 P_2 P_1, using for this purpose a pattern of the joint.

It will then be easy to dress the face which shows upon the battered surface, since it is a plane passing through the three right lines R_1 M_1, M_1 N_1, N_1 P_1, which are known, and by means of a pattern the contour M_1 N_1 P_1 Q_1 R_1 can be traced. This having been done, and the arc M_1 N_1 being then known, cut out the cylindrical intrados by using a templet cut to the curve M'N' (Fig. 66). Applying now to this cylindrical surface the pattern of the developed intrados, trace the arc M_2 N_2 which limits the intrados at the end.

It remains only to cut the two plane faces R_1 Q_1 Q_2 R_2 and P_1 Q_1 Q_2 P_2 and the curved end of the voussoir. The plane faces are easily cut,

since for each of them two right line directrices are known. Upon the edges $R_1 R_2$ and $Q_1 Q_2$ of the first of these faces take $R_1 R_2$ = R R' and $Q_1 Q_2$ = Q Q', and join R_2 and Q_2 by a right line, which ought to be perpendicular to the edges. Similarly, upon the edges $Q_1 Q_2$ and $P_1 P_2$ of the second face take $Q_1 Q_2$ = Q Q" and $P_1 P_2$ = P P", and through Q_2 and P_2 pass a curve, to be traced by means of a templet cut to the curvature of a right section of the larger arch. The remaining end face of the voussoir, a cylindrical surface of which the contour $M_2 N_2 P_2 Q_2 R_2$ is completely determined, can be cut by means of a straight-edge kept always parallel to the edge $R_2 Q_2$.

26. Remarks:- The disadvantage of having stones with acute angles has already been noticed. Now it will be seen in Fig. 66 that the horizontal surfaces of the voussoirs form with their end faces lying in the large arch angles which are the more acute as the voussoir in question is nearer the key and as the radii of the two arches approach each other. This difficulty can be met by terminating these horizontal faces by a short face normal to the large arch.

It is to be noticed also that the angle E C C" which the vertical face C C" of the first voussoir forms with the slope is the more acute as the obliquity of the wall increases. This voussoir might therefore be given a narrow face perpendicular to F. C. The same thing might be done for the other voussoirs at P, D, etc., although it would be plainly objectionable to do it at F. In case the skew were considerable resort would sometimes be had to the elbow arch, composed of two arches, one perpendicular to D C and the other to D"C".

If the large arch were of cut stone, then the voussoirs in the smaller arch should conform to the courses of the larger, and would become somewhat complicated, each voussoir at the intersection having two intradosal surfaces, one for each arch. A different drawing would then be required, and the problem would come under the head of Lunettes, to be considered later.

27. An Arched Opening in a Cylindrical Wall or Round Tower:-

Let the circular arcs A e B and C f D (Fig. 69) be the horizontal traces of the cylindrical wall or round tower, and let it be required to construct in this wall an arched opening to be comprised between two parallel planes A C and B D which are perpendicular to the chord C D of the arc intercepted by the opening. The intrados of the arch will be projected vertically along a curve A'U'B', here taken as a semi-circle. Divide this curve into an odd number of equal parts, five for example, and through the points of division and through the axis O'f of the cylinder of the intrados pass planes M'O'f, N'O'f, etc., which will divide off the voussoirs and will cut the intrados along rectilinear elements. Terminate the joints thus formed, M'R', N'P', etc., by a circle E'R'F' concentric with the first, and limit each voussoir by a horizontal and a vertical face such as R'Q' and Q'P'.

One of the voussoirs will be projected entirely upon the polygon M'N'P'Q'R', and will occupy in the right prism having this polygon for a base the space comprised between the cylindrical surfaces which form the faces of the round tower. The horizontal projection of this and the other voussoirs will be obtained by dropping from the various vertices of the pentagons, such as M'N'P'Q'R', verticals M'M M", N'N N", etc., of which the parts M M", N N", etc., contained between the two faces of

the cylindrical wall will be the horizontal projections of the edges of the joints. Since the faces of the wall are vertical cylinders, the heads of the voussoirs will be projected horizontally upon the traces A e B and C f D.

28. <u>Patterns</u>:- Patterns of the joints and of the intradosal surfaces will be required. The latter are parts of a cylindrical surface whose right section is A'U'B', and will evidently be shown in their true size if that surface be developed. Rectify, therefore, the curve of right section, by taking upon an indefinite right line X Y (Fig. 70) distances b n, n m, etc., equal to the arcs B'N', N'M', etc.,(Fig. 69), which is done by stepping off small chords as many times as necessary. Then, having erected perpendiculars to X Y through the points of division, take b b' = g B, n n' = h N, m m' = i M, ... a a' = k A. Through the points b', n', m', ... a' thus obtained pass a curve b'n'm'a', which will be the development of the curve of the head projected in A e B, which development can be made more precisely by finding intermediate points between those above indicated. Similarly, take b b" = g B", n n" = h N", m m" = i M", ... a a" = k A", and through the points b", n", m", ... a" thus obtained pass a second curve b"n"m"a", which will be the development of the curve of the head of the arch upon the inner face of the round tower. The true form and size of all the intradosal surfaces are now perfectly known and will be represented by the quadrilaterals b'n'n"b", n'm'm"n", etc.

The joints are plane surfaces and will be obtained by taking upon X Y distances n p = N'P', m r = M'R', etc., and erecting through each of the points p, r, etc., thus obtained, perpendiculars p p' = l P, r r' = u R, etc.; p p" = l P", r r" = u R", etc. As the intersectionsof the joints with the inner and outer cylindrical faces of the round tower are elliptical curves, it will be necessary also to determine at least one intermediate point between the extreme points found above, which may be done by applying the same procedure to horizontal right lines (Fig. 69) passing, for example, through the middle of the joints N'P', M'R', etc.

29. <u>Cutting the Voussoirs by the Method of Squaring</u>:-

Choose a block of stone having a length at least equal to the greatest dimension P v (Fig. 69) of the horizontal projection of the voussoir to be cut. The other dimensions of the block should be such that a pattern of the head M'N'P'Q'R' could be applied upon the two end faces. Begin by dressing one of the bases to a plane, and trace upon it the contour M_1 n_1 p_1 q_1 r_1 (Fig. 71) by means of a pattern cut to the vertical projection M'N'P'Q'R' (Fig. 69), taking care that the side M'R' of one of the joints coincide as nearly as possible with the quarry bed m_1 r_1 R_2 M_2.

Then cut away the stone, square with the base already dressed, along the line M_1 r_1, and to the new face apply the pattern m'r'r"m" (Fig. 70) of the upper joint, placing it so that m' falls at M_1 and so that r' is distant from r_1 by a length r_1 R_1 = R z (Fig. 69). By means of this pattern trace the contour M_1 R_1 R_2 M_2 of the joint. Proceed in the same manner for the joint which passes through the right line n_1 p_1 and which should be square with the base, and apply to it the corresponding pattern n'p'p"n" (Fig. 70) in order to trace the contour of the lower joint.

The intrados is a portion of a cylinder passing through the curve M_1 n_1 and whose elements are perpendicular to the base M_1 n_1 p_1 q_1 r_1.

The square will answer for cutting this cylinder, or if its branches are too short a templet may be used, cut to the arc M'N' (Fig. 69), and moved along the right lines already traced, $m_1 M_2$, $n_1 N_2$, in such a manner as to be always parallel to the base M'n'p'q'r'. Next apply the pattern of the intrados m'n'n"m" (Fig. 70) to the concave surface of the stone and trace the two curves $M_1 N_1$ and $M_2 N_2$.

The lateral face $p_1 q_1 Q_2 P_2$ and the upper face $q_1 r_1 R_2 Q_2$ are planes, each of which passes through two known right lines, and are also perpendicular to the base of the prism; they can therefore be easily cut by aid of the straight-edge. That done, take the distances $q_1 Q_1$ and $q_1 Q_2$ respectively equal to the lengths v P and v P" (Fig. 69) and trace the right lines $Q_1 P_1$, $Q_2 P_2$, as well as the curves $Q_1 R_1$, $Q_2 R_2$, using for the latter purpose templets cut to the arcs P R, P"R" of the traces of the round tower.

It remains only to cut the two heads of the voussoir, of which we now know the contours $M_1 N_1 P_1 Q_1 R_1$ and $M_2 N_2 P_2 Q_2 R_2$. These surfaces, being cylindrical, can be cut by moving a straight-edge upon the contours in such a manner as to pass through points of reference suitably chosen; or in such a manner as always to be parallel to the face $P_1 Q_1 Q_2 P_2$.

It is possible to cut the stone without using the constructions of Fig. 70, by noting upon a sketch of the stone the lengths of the different right lines N x, N"x, R z, R"z, P v, P"v, etc., and of some other intermediate lines; then, having squared a right prism having for its base the contour $M_1 n_1 p_1 q_1 r_1$ (Fig. 71), mark upon the stone the distances $r_1 R_1$, $r_1 R_2$, $n_1 N_1$, $n_1 N_2$, etc., respectively equal to the corresponding distances given on the sketch, and finally trace by hand the curves $M_1 R_1$, $R_1 Q_1$, $M_1 N_1$, etc., for which one or two intermediate points will be needed.

If the diameter of the arch differ but little from that of the round tower, the angles which the vertical faces of the voussoirs make with the inner face of the tower are likely to be too acute, especially towards the springing lines of the arch. This difficulty may be avoided by making the vertical faces perpendicular to the faces of the tower, for a few inches in, as indicated for the face F F" (Fig. 69), c d being any direction whatever.

It has here been assumed that the round tower is of brick masonry, and that no care need be taken to make the joints of the voussoirs accord with the courses of the tower. But if the latter were of stone, the joints of the voussoirs might not stop at a circle concentric with that of the intrados, but might be given sufficient length to bring about the accordance in question. One of the arrangements represented in Fig. 62 would probably be adopted, but the cutting of the stones would present no new difficulty.

30. A Skew-Arch in a Round Tower with Battered Face and Meeting a Spherical Dome:-

It is assumed that the exterior face of the tower is that of a cone of revolution, or that, in other words, it has a batter, the value of which is indicated by the angle Z'C'Y' (Fig. 72), and that the tower is covered by a spherical dome. An arched opening is to be constructed in the wall of the tower, having its springing plane on the same level with that of the spherical dome. The arch is to be on a skew, that is,

its axis is not to meet that of the tower.

The vertical plane of projection will be taken perpendicular to the axis of the arch, and upon this the semi-circle A'M'B' will represent the right section of the intrados of the arch. The horizontal plane will be taken in the common springing plane of arch and dome. Let E A B be the trace of the cone upon the springing plane, and F A"B" the face of the cylindrical inner face of the tower; it will at the same time be the horizontal great circle of the dome.

Divide the semi-circle A'M'B' into an odd number of equal parts, five for example, and indicate the joints by right lines R'M'O', P'N'O', etc., all passing through O' in order that they may be normal to the intrados. Limit these joints N'P', M'R', etc., by a circle C'R'D' concentric with A'M'B', and terminate each voussoir by a vertical face P'Q' and a horizontal face Q'R'.

The round tower is assumed to be built of brick masonry; but if it were of cut stone it would perhaps be necessary to modify the construction somewhat and to arrange that the horizontal face of each voussoir should be in the same plane with one of the beds of the courses composing the tower. As it is, each voussoir is vertically projected upon a pentagon such as M'N'P'Q'R' and is part of a right prism having one of these pentagons for a base; further, it occupies in the prism the space between the spherical dome, on the one hand, and the exterior face of the round tower, on the other hand. It is necessary, then, to determine the intersection of this prism with the sphere and with the cone.

In order to determine the intersection with the cone, imagine through the point of which we wish to know the horizontal projection, that for example which is vertically projected at M', a horizontal circle, lying upon the conical surface. This circle will be projected vertically along a right line parallel to the ground line and meeting C'Y' at a point m', which we project horizontally to m upon C Y drawn parallel to the ground line. Then revolve this point into the plane C C" by means of an arc of a circle m m", and through the point m" thus obtained pass a circle m"M having the same centre as the round tower. The intersection of this circle with the perpendicular dropped from M' will furnish the horizontal projection M of this point.

It will be noticed that the radius of the circle m"M differs from that of the circle E A B by a distance equal to m_1 m'. Consequently it is not necessary, in order to find the point M, to proceed as above indicated; but it will be sufficient to draw any right line E F G passing through the center of the tower, to take the distance E m_2 = m'm_1, and to pass through m_2 an arc having the same center as the tower, which arc will evidently be the same as the arc m M previously determined. Similarly one will find the horizontal projections of the other points where the edges of the voussoirs meet the outer surface of the tower, thus determining the curve B N M A. The horizontal face Q'R' will cut the cone along a circumference having the same center as the round tower; consequently the arc Q R, which is the prolongation of that which furnished the point R, will be the projection of this intersection. The joints N'P' and M'R' meet the cone along curves P N O and R M O which pass through O and on which intermediate points can be found by means which have been indicated. Finally, the vertical face P'Q' will cut the cone along a curve projected upon the right line P Q.

In order to construct the head of the voussoir which is upon the sphere, draw horizontals through M' and N' and describe upon the horizontal

plane two circumferences m_4 M" and n_4 N" concentric with A"O"D" and having for radii the radius of the latter diminished by the distances m_3 g and n_3 h comprised between the vertical D'V' and the arc D'U', which is a section of the spherical dome. The same thing will be done for the intradosal edges of the other voussoirs, thus giving the curve A"M"N"B" along which the intrados of the arch meets the spherical dome.

As for the joint M'R', the horizontal edge through the point R' will pierce the sphere at a point of which the projection R" will be obtained by drawing the horizontal R'k r_3 and subtracting the part k r_3 from the radius of the arc A"O"D", describing with the remainder an arc which will cut the perpendicular let fall from R' at the desired point R". The intersection made by the joint itself will be projected along a curve R"M"O", of which an intermediate point can be found by applying the preceding method to the middle of the side M'R'.

An analogous curve P"N" will be found for the lower joint; the horizontal face R'Q' will give the arc R"S", prolonged from that which has served to determine the point R"; and finally the vertical face P'Q' will cut the sphere along an arc projected upon the right line P P". But as this last face meets the sphere very obliquely, pass through P" a plane P"S" passing also through the axis of the round tower, thus reducing the projection of the head of the voussoir to M"N"P"S"R", and giving rise to a new vertical face which is shown revolved in the triangle a b c. In this triangle, which is right-angled, the hypothenuse b c is an arc coincident with A"B"D" prolonged, and the height a b is equal to that of the vertical face of the voussoir, that is, to P'Q'.

The vertical face being thus modified it is necessary, in cutting the stone, to know its exact shape. Therefore revolve it into the horizontal plane, giving the figure p'p"q"q', of which the height p"q" is equal to P'Q', and of which the side p'q' is a curve easily constructed by erecting perpendiculars from different points of P Q and laying off upon these, from p'p", lengths respectively equal to the distances which, upon the vertical plane, separate the corresponding points from P'.

31. <u>Making the Patterns</u>:- The development of the cylindrical intrados will be effected by taking upon an indefinite right line distances b n, n m, etc., equal to the arcs B'N', N'M', etc.; then, having erected perpendiculars through the points of division, take b b' = e B, n n' = f N, m m' = i M ..., a a' = j A, and the curve b'n'm'a' will be the development of the curve projected in B N M A. This can be obtained more precisely by procuring intermediate points between b', n', m', ... a'. In the same way take the distances b b" = e B", n n" = f N", m m" = i M", ..., a a" = j A", and the curve b"n"m"a" will be the development of the curve in which the intrados of the arch meets the spherical dome. Then the exact form of the intradosal surfaces will be known, and will be represented by the quadrilaterals b'n'n"b", n'm'm"n", etc.

The bed joints are plane surfaces, and their patterns will be obtained by laying off on the development distances n p = N'P', m r = M'R', etc., and erecting through each of the points p, r, etc., thus obtained, perpendiculars p p'= y P, r r' = z R, etc.; p p" = y P", r r" = z R", etc. As the edges of the joints, upon the conical and spherical surfaces, are curves, it will be well to obtain at least one intermediate point upon each, as by applying the above method to the middle points of the joints N'P', M'R', etc.

32. Cutting the Voussoirs:- Begin by forming a right prism, having a base $M_1\ n_1\ p_1\ q_1\ r_1$ (Fig. 74) identical with the contour M'N'P'Q'R' (Fig. 72); and a length at least equal to the distance of S" from the line M s. Arrange that one of the joints coincide with the quarry bed. Then, upon the face which passes through the side $n_1\ p_1$ apply the pattern n'p'p"n" (Fig. 73) of the lower joint in such a manner that $n_1\ N_1$ = u N, and $p_1\ P_1$ = s P. Similarly, upon the face which passes through $M_1\ r_1$ trace the contour $M_1\ R_1\ R_2\ M_2$ by means of the pattern m'r'r"m" (Fig. 73). Upon the upper face, which passes through $q_1\ r_1$, indicate the contour $Q_1\ R_1\ R_2\ S_2\ Q_2$ by using a pattern cut to R Q P"S"R" (Fig. 72).

Then, through the two known right lines, $P_2\ Q_2$, $Q_2\ S_2$, pass a plane, upon which trace the contour $P_2\ Q_2\ S_2$ by means of the pattern a b c (Fig. 72); after which apply upon the cylindrical face cut along $m_1\ n_1$ the intradosal pattern m'n'n"m" (Fig. 73) and trace the curves $M_1\ N_1$, $M_2\ N_2$ which limit the intrados. Finally, upon the face passing through the right line $p_1\ q_1$ mark the contour $P_1\ Q_1\ Q_2\ P_2$ by means of the pattern p'q'q"p" (Fig. 73), taking care that $p_1\ P_1$ = s P and $q_1\ Q_1$ = s Q.

The intrados and all the other lateral faces having been cut, and the contour $M_2\ N_2\ P_2\ Q_2\ R_2$ of the head of the voussoir being then known, that head, which is spherical, can then be cut by employing a templet cut to the curvature of D'U' (Fig. 72) applying it always in a direction perpendicular to the face $R_1\ R_2\ S_2\ Q_2\ Q_1$.

As for the exterior head, of which the contour $M_1\ N_1\ P_1\ Q_1\ R_1$ is also completely known, it is a portion of a conical surface and can be cut by aid of the straight-edge applied in the direction of elements. These can be determined by fixing upon the drawing reference points, conveniently spaced, which reference points can be obtained by drawing from the center of the round tower right lines such as v x (Fig. 72).

THE DESCENT, OR RAMPANT ARCH.

33. A descent, or rampant arch, is an arch the elements of whose cylindrical intrados are inclined to the horizontal plane, instead of being horizontal as in the ordinary arch. This difference in the position of the elements leads to other differences in the form of the arch and sometimes gives rise to considerable difficulties in the drawing.

34. A Descent through a Right Wall:- In Fig. 78 let A'M'B' be the intersection of the intrados with a vertical plane A'B' which is one of the faces of the wall. Moreover let B'B" be the base of the ramp, the height of which is given. This base and this height form a right-angled triangle, the hypothenuse of which is the direction of the elements of the oblique cylinder which has for a base the circle A'M'B'. In order to show better this direction take a profile or vertical plane $B_2\ B_3$ parallel to B'B", and having revolved it upon the horizontal plane around $B_2\ B_3$ trace there the right-angled triangle $B_2\ B_3$ C, the hypothenuse of which will show the true slope of the ramp. It is clear that this hypothenuse B_3 C will be the trace of the springing plane upon this profile plane, and that A"B_3 will be the horizontal trace. The two ramps of the

descent are rectangles, projected upon A'E'E"A" and B'F'F"B".

Divide the face curve into an odd number of equal parts, five for example, and through the points of division and the center O' pass inclined joints and limit these by a circle E'R'F' concentric with the first, finally terminating each voussoir by a horizontal face and a vertical face such as R'Q' and P'Q'. Pentagons such as M'N'P'Q'R' will thus be obtained which are the upper heads of the voussoirs composing the descent. The horizontal projections of these same voussoirs are made up of the projections of the various parallel edges. In order to determine the projections of these edges upon the profile plane, through the points M', N', P', etc., draw horizontals M'm, N'n, P'p, etc., and through the points m, n, p, etc., where they meet the vertical C Z, pass arcs having a common center at C. These arcs will cut the line B_2 C D at points m_1, n_1, p_1, etc., through which draw, parallel to the rampant line C B_3, right lines m_1 m_2, n_1 n_2, p_1 p_2, etc., which will be the projections of the intradosal and joint edges upon the profile plane, which projections evidently show them in their true length.

In order to develop the intradosal surfaces for patterns, it will be necessary to know the right section of the descent. To this end draw through any point c_1 of C B_3 a perpendicular C_1 r_3 T. This perpendicular will cut the edges n_1 n_2, p_1 p_2, etc., at points n_3, p_3, etc., which are to be revolved upon the vertical c_1 V by means of arcs n_3 n_4, p_3 p_4, etc. Through the points n_4, p_4, etc., thus obtained, and through the point c_1 draw horizontals, meeting the horizontal projections of the edges of the voussoirs at points M_1, N_1, P_1, etc. Then trace the curve A_1 M_1 B_1 which will be a right section of the descent, and join by right lines the points such as M_1 and R_1, N_1 and P_1, R_1 and Q_1, etc., which will give a pattern for obtaining the heads of the voussoirs.

35. <u>Obtaining the Patterns</u>:- In order to rectify the cylinder of the intrados, lay off upon the indefinite right line S T (Fig. 79) distances S n, n m, etc., equal to the lengths of the corresponding arcs B_1 N_1, N_1 M_1, etc., of the right section (Fig. 78); then erect perpendiculars, upon which take distances S b' = c_1 C, n n' = n_3 n_1, m m' = m_5 m_1, etc.; and S b" = c_1 B_3, n n" = n_3 n_2, m m" = m_3 m_2, etc. It will then be easy to trace the curves b'n'm'a' and b"n"m"a", which will be the developed face curves of the rampant arch, and which will be identical with each other, since the arch is in a right wall. The true size and exact form of all the intradosal surfaces will be perfectly known and will be represented by quadrilaterals b'n'n"b", n'm'm"n", etc.

As for the patterns of the joints, which are plane surfaces, they may be obtained by laying off upon the right line S T, from the intradosal edges, distances n p = N_1 P_1, m r = M_1 R_1, etc., and erecting through each of the points p, r, etc., thus obtained, perpendiculars p p' = p_3 p_1, r r' = r_3 r_1, etc.; p p" = p_3 p_2, r r" = r_3 r_2, etc. The right lines n'p', n"p", m'r', m"r", etc., will be the exterior edges of the joints, that is to say, those which limit them upon the faces of the wall. (Certain lines shown in the drawing, but not mentioned, relate to a descent to be studied later).

36. <u>Cutting the Voussoirs</u>:- Assume that it is desired to cut the voussoir which has for its head M'N'P'Q'R' (Fig. 78). Begin by squaring a right prism having a base m_2 N_2 p_2 q_2 r_2 (Fig. 80) identical with the contour M_1 N_1 P_1 Q_1 R_1 (Fig. 78) and having a length at least equal to the distance of the point n_2 from the right line u r_1. Arrange so that

one of the joints shall coincide with the quarry bed of the stone. Then take upon the prism distances $m_2 M_2$, $p_2 P_2$, $q_2 Q_2$, $r_2 R_2$, equal to the corresponding distances on the drawing, that is to say, to $m_2 z$, $p_2 y$, $r_2 s$, the last answering for both $q_2 Q_2$ and $r_2 R_2$.

Then, upon the cylindrical face which passes through the arc $m_2 N_2$ apply the intradosal pattern m'm"n"n' (Fig. 79) and trace the contour $M_2 N_2 N_1 M_1$. Apply in the same manner upon the faces which pass through the right lines $m_2 r_2$ and $n_2 p_2$ patterns m'r'r"m" and n'p'p"n" (Fig. 79) and trace the contours $M_2 R_2 R_1 M_1$ and $N_2 P_2 P_1 N_1$. Finally take the distances $R_2 R_1$, $Q_2 Q_1$, $P_2 P_1$, equal to the lengths of the corresponding edges measured upon the drawing, edges which are evidently of the same length, since, the wall being a right wall, its two faces are parallel.

The contours of each of the heads being then known, it will be easy to cut their surfaces, which are plane. To this end it is only necessary to cut away the two truncated prisms which are comprised at each end between the base and the adjacent head. The intrados, a cylindrical surface, will be cut by aid of a straight-edge made to pass through reference points suitably chosen upon the face curves, and which can easily be obtained from the drawing.

37. Remarks:- It will be seen from the above directions that patterns of the intrados and joints are not essential to the cutting of the voussoir.

The joints of the arch, as above drawn, are oblique to the intrados, which may be of considerable importance if the slope of the descent is great. If it is desired to make them normal to the intrados, proceed as follows: Construct the right section $A_1 M_1 B_1$ (Fig. 78), and after having drawn true normals $N_1 P_1$, $M_1 R_1$, terminate them at points P_1, R_1, where they meet horizontals from p_4 and r_4; then by the reverse of the operations hitherto described obtain from P_1 and R_1 the points P' and R', which will fix the direction of the edges of the joints N'P' and M'R' upon the plane of the head. It is true that these lines will not be normal to the face curves, but this is thought not to be so serious an objection as to condemn the plan proposed.

If the descent were of considerable length and it were no longer possible to form each course of a single stone, it would be unwise to allow the whole mass of the arch to rest upon two inclined surfaces such as the ramps which in Fig. 78 are projected in the rectangles A'A"E"E' and B'B"F"F', because there would be danger of the mass sliding, or at least seriously straining by its pressure the arch or other work against which it might abut. For this case, instead of terminating the abutments by an inclined plane, prolong the voussoirs of the first course down into the abutment, where they are to rest upon horizontal beds, as seen in Fig. 81, which is a section along the axis in an arch thus arranged. In order to guard still further against sliding, the voussoirs may be given a form such as a'b'c'd'e'f'g'h'i', by which means they are tied together. Some simple stones are employed, however, as indicated by diagonals in Figs. 81 and 82, the latter representing a projection of the intrados upon a plane parallel to the elements. Such stones are employed for the key course, which are the last to be cut and are carefully fitted to the space left between the adjacent courses. Figure 83 illustrates the general form of the voussoirs in this construction.

The arrangement above outlined prevents sliding, but does not prevent the intrados and joints from meeting obliquely the planes of the two faces. It is therefore an advantage to begin and to end the descent with a short horizontal arch, as indicated in Fig. 84.

38. A Right Descent in a Battered Wall, meeting an Arch in Brick Masonry:-

The same data will be adopted as for Fig. 78; but as the descent is here supposed to meet a large masonry arch, the springing plane of which coincides with the horizontal plane of the drawing, and the first element of which is E"F", it will be necessary, in order to define the arch, to trace the arc B_3 X with a given radius, which will represent a right section of the cylinder made by the vertical plane B_2 B_3. Further, in order to complete the data, trace upon the profile plane the right line C Y, directing it relatively to the vertical so as to show the batter of the wall.

Then begin by determining the face curve A' i g B' as follows:- Through the point e, where, upon the profile plane, the right line C Y meets the edge of the intrados n_1 n_2, drop a perpendicular e f upon C D and through the point f pass an arc f h having its center at C. This arc meets the vertical C Z at a point h, through which draw a horizontal cutting at g the vertical from N'. The point g is one point of the face curve, and other points may be found in the same manner.

The right section will be obtained exactly as explained in Article 34, and the intradosal and joint patterns as in Article 35. The patterns will differ, however, a little from those of Fig. 79, since in order to obtain them one will no longer lay off from S T the distances comprised between c_1 r_3 and C D, B_3 r_2, but instead those comprised between c_1 r_3 and the right line C Y and the arc B_3 X.

The cutting of the voussoirs offers no new feature, except that the head lying in the masonry arch will be cylindrical instead of plane, and will be executed as explained in Article 36.

39. A Skew Descent meeting an Arch in Brick Masonry:- In Fig. 85 let A'M'B' be the face curve of the descent, which we suppose situated in the vertical plane of the drawing; the right lines A'A", B'B", the horizontal projections of the springing lines of the intrados of the descent; C"D", the first element of the large arch which the descent is to meet, and the springing line of which is in the horizontal plane of the drawing; and, finally, let C" m_3 X be the right section of the large arch, revolved into the horizontal plane.

First, let there be determined upon the vertical plane which has C"E for its horizontal trace the projections of the different intradosal and joint edges of the voussoirs which form the descent, the joints first having been fixed in the customary manner by dividing the curve A'M'B' into an odd number of equal parts, five for example, and drawing through the points of division right lines such as N'P', M'R', which pass through the axis of the descent.

For this purpose, from the point E lay off a length E b_2 (Fig. 86) equal to the height of the slope, and join b_2 with C": the right line b_2 C" will be the common projection of the two intradosal edges A'A" and B'B" upon this profile plane. To obtain the projections of the other edges of the voussoirs, draw through the points M', N', P', R', etc., horizontals N' n_1, M' m_1, P' p_1, R' r_1, etc., and through the points n_1, m_1, p_1, r_1, etc., where these horizontals meet the vertical b_2 Z, pass arcs having for a common center the point b_2. These arcs will meet the line D'C' prolonged at points n_2, m_2, p_2, etc., through which draw, parallel to b_2 C", right lines meeting the right section curve of the large arch at points n_3,m_3,p_3,etc.Finally, through these new points draw horizontals, which cut the horizontal projections of the edges of the voussoirs at points N", M", P", etc., which will evidently be the horizontal projections of the points where these edges meet the large arch. Tracing then the curve A"M"N"B" there will be given the horizontal projection of the intersection of the intrados of the descent with that of the large arch.

That done, it will be noticed that the faces such as R'Q' must necessarily meet the large arch along elements such as R"Q". Further, the vertical faces such as P'Q' will cut the large arch along curves; but these curves, since the faces are vertical, will be projected upon right lines such as P"Q". The joints will meet the large arch along curves P"N"O", R"M"O", etc., which should pass through the point O" and be tangent at that point to the right line C"D".

The vertical projection (Fig. 86) does not give, as in the case of a right descent , the true lengths of the edges of the voussoirs, since the ground line C'E is not parallel to the horizontal projections of these edges. It is necessary, in order to find those true lengths, to procure a second vertical projection on a plane parallel to the horizontal projecting planes of the edges. It is necessary, moreover, to determine the vertical projection of Figure 86 in order to be able to find the horizontal projections of the faces of the voussoirs upon the large arch, which projections will shortly be useful.

In order to obtain the second vertical projection (Fig. 87), take a ground line D'D" parallel to the horizontal projection O'O" of the axis of the descent; then, after having marked the height D' D_1 of the descent, draw the horizontal D_1 C_1 upon which project the points B', A', C', to B_1, A_1, C_1, and project upon the ground line the points B", A", C", to B_2, A_2, C_2. Then the right lines D_1 D", B_1 B_2, A_1 A_2, C_1 C_2 will be the projections of the sides of the two ramps; the right line D_1 C_1 will be the vertical projection of the intersection of the plane which passes through the springing elements of the descent with the vertical face of that descent; and the ground line D'D"C_2 will be the vertical projection of the intersection of the same springing plane with the large arch, which intersection is at the same time the springing line of that arch.

In order to find the vertical projection of one of these edges, that, for example, which starts from the point M' (Fig. 85), draw through this point the horizontal M' m_4, which will meet the vertical F U at m_4; then through m_4 pass an arc described from F as a center and meeting at m_5 the right line F V perpendicular to the right line F D_1 C_1; then, through this new point m_5 draw a parallel to D_1 C_1, which will cut at M_1 the right line M I M_1 perpendicular to the ground line D'D"C_2; finally, through

the point M_1 draw the right line $M_1 M_2$ parallel to $B_1 B_2$, and the point M_2, where this parallel meets the perpendicular $M"G M_2$ to the ground line $D'D" C_2$, will be the vertical projection of the point where the edge in question pierces the intrados of the large arch. Proceed in the same manner for the intradosal edge which starts from N'; so that the curves $B_1 N_1 M_1 C_1$, $B_2 N_2 M_2 C_2$ will be the lateral projections,- the one, of the face curve B'N'M'C', and the other, of the intersection of the intrados of the descent with that of the large arch. The edges of the extrados, such as those starting from the points P', Q', R', will be obtained in a similar manner.

The joint lines upon the plane of the head will be projected along right lines $P_1 N_1$, $R_1 M_1$, which should pass through the point O_1; and those upon the large arch, along curves $P_2 N_2$, $R_2 M_2$, which ought equally to meet at O_2. The lines, such as $P_2 Q_2$, proceeding from vertical faces, will be curves intermediate points of which can be obtained by means already indicated; the other lines, such as $P_1 Q_1$, proceeding from vertical faces, will be right lines perpendicular to the ground line $D'D"C_2$; finally, lines such as $R_1 Q_1$ and $R_2 Q_2$ will be right lines parallel to the ground line $D'D"C_2$.

In order to obtain a right section of the descent, draw the right line H K (Fig. 87) perpendicular to the projections of the elements of the intrados of the descent; then, after having prolonged (Fig. 85) to a convenient length the horizontal projections of the edges of the intradosal surfaces and of the extrados, draw the right line S T perpendicular to those projections; afterward, through the point B_3 where S T meets the edge B'B" produced pass a line $D_3 C_3$ such that the distance u A_3 shall be equal to c d (Fig. 87), and $B_3 A_3$ will be the diameter of the right section. Then make the distances v M_3 = c f, x N_3 = c e, etc., and through the points A_3, M_3, N_3, B_3 draw the curve A_3 ... $M_3 N_3 B_3$ which will be the required right section.

The joint lines will be obtained by making the distances y R_3 = c h, z P_3 = c g, etc., and drawing the right lines $M_3 R_3$, $N_3 P_3$, etc.,which should necessarily converge to the point O_3. Finally, the upper and lateral edges such as $R_3 Q_3$ and $Q_3 P_3$ will be obtained by drawing parallels $R_3 Q_3$ to the diameter $A_3 B_3$, and drawing other parallels $Q_3 P_3$ to the axis $O'O_3$ of the descent. It will be noticed that the distance Q_3 z should equal i c.

40. Patterns of the Development:- Upon an indefinite right line (Fig. 89) take the lengths $b_6 n_6$, $n_6 m_6$, etc., equal to the arcs $B_3 N_3$, $N_3 M_3$ of Fig. 88, and erect the perpendiculars $b_6 b_7$ and $b_6 b_8$, $n_6 n_7$ and $n_6 n_8$, $m_6 m_7$ and $m_6 m_8$, etc., respectively equal to the distances c B_1 and c B_2, e N_1 and e N_2, f M_1 and f M_2, etc. The curves $b_7 n_7 a_7$ and $b_8 n_8 a_8$ will be the developments of the two face curves of the descent, and will determine at the same time the patterns of the intradosal surfaces such as m $n_7 n_8 m_8$.

In order to obtain patterns of the joints, take the distances $n_6 p_6$, $m_6 r_6$, etc., respectively equal to $N_3 P_3$, $M_3 R_3$, etc., (Fig. 88), and erect the perpendiculars $p_6 p_7$ and $p_6 p_8$, $r_6 r_7$ and $r_6 r_8$, etc., equal to the lengths g P_1 and g P_2, h R_1 and h R_2, etc. (Fig. 87). The sides $p_7 n_7$, $r_7 m_7$, etc., of these patterns will be right lines, and the sides $p_8 n_8$, $r_8 m_8$, etc., will be curves, of which intermediate points can be secured by applying the same procedure to the middle point of the sides N'P', M'R', etc., of Fig. 85.

41. Cutting the Voussoirs:- Suppose that it is required to cut the voussoir which has for a face M'N'P'Q'R' (Fig. 85). Begin by squaring a right prism having for a base the contour M_3 N_3 P_3 Q_3 R_3 (Fig. 88), and upon the intrados of this prism apply the pattern m_7 n_7 n_8 m_8 of Fig. 89, by means of which trace the contour M^1 N^1 N^2 M^2, arranging that the distances n^1 N^1 and m^1 M^1 be equal to j N_1 and s M_1 (Fig. 87). Similarly, upon the faces which pass through the right lines m^1 r^1 and n^1 p^1 apply the patterns m_7 r_7 r_8 m_8 and n_7 p_7 p_8 n_8, and trace the contours M^1 R^1 R^2 M^2 and N^1 P^1 P^2 N^2, making the distances r^1 R^1 and p^1 P^1 respectively equal to k R_1 and t P_1 (Fig. 87). Finally, upon the face which passes through Q^1 p^1 apply a pattern cut along P_1 Q_1 Q_2 P_2 (Fig. 87), which is the true size of the vertical face of the voussoir, since this face is parallel to the plane of projection; and by means of this pattern indicate the contour P^1 Q^1 Q^2 P^2. Then draw the right lines Q^1 R^1, Q^2 R^2, and the two faces will be completely defined. The first of these, being plane, will easily be cut; the second, which is upon the large arch, being a portion of a cylinder having its elements parallel to R^2 Q^2, will be cut by the aid of a straight-edge kept always parallel to R^2 Q^2, or made to pass through reference points conveniently chosen.

CONICAL VAULTS.

42. The conical vault has its intrados a conical surface. For the purpose of this problem let there be given a right wall, whose faces have for horizontal traces the right lines A B and C D (Fig. 91), in which is to be constructed an opening covered by a vault whose intrados is to be a right cone having its apex S upon the springing plane, and having for directrix the curve A'M'B' traced upon the face A B of the wall, a curve which will in this case be taken as a semi-circle.

It follows from these assumptions that the apex of the cone will be vertically projected at the center O' of the face curve A'M'B', and that the cone will cut the second face C D of the wall along a semi-circle C'm'D', of which the diameter C'D' will be equal to the length C D comprised between the two springing lines A C S, B D S.

Begin by dividing the semi-circle A'M'B' into an odd number of equal parts, five for example. Then, through the points of division drop perpendiculars upon the ground line, which will meet the trace A B of the face containing the directrix of the cone at points M, N, etc., which, joined to the apex S, will furnish the horizontal projections M m, N n, etc., of the intradosal edges. The vertical projections M'm', N'n', etc., of these same edges will be obtained by joining the center O' to the points of division M', N', etc.; then, prolonging these lines by a suitable amount, the vertical projections M'R', N'P', etc., of the joints will be obtained. Finally, terminate each voussoir by a vertical face Q'R' and by a horizontal face P'Q', and the drawing will be complete.

43. Cutting the Voussoirs:- Take for example the voussoir which is vertically projected in the pentagon m'n'P'Q'R' (Fig. 91). Begin by squaring a right prism having this pentagon for a base, which will give a stone having the form m_1 n_1 P_1 Q_1 R_1 R_2 Q_2 P_2 n_2 m_2 (Fig. 92). Then,

upon the front face mark two points M_1 and N_1 such that the distances $M_1 R_1$ and $N_1 P_1$ are respectively equal to the lengths M'R' and N'P' of Fig. 91. Through the points M_1, N_1 pass an arc traced by means of a templet cut to the arc M'N' of Fig. 91, and then draw $M_1 m_2$ and $N_1 n_2$. The intrados, which is a conical surface, may now be cut by the aid of a straight-edge resting continually upon the arcs $M_1 N_1$ and $m_2 n_2$ and passing through reference points suitably chosen. The lines u v, x y show the direction which the straight-edge should receive in two of its positions, which evidently correspond to two elements of the cone to which the intrados belongs. The reference points u, v, x, y, are easily determined by taking the arcs M_1 u, u x, x N_1, m_2 v, v y, y n_2, respectively equal to the arcs M'u, u x, x N', m'v, v y, y n' (Fig. 91).

44. Vertical Conical Vault:- This name is given to a vault the intrados of which is a right circular cone with vertical axis. The construction may be used to cover a cylindrical tower in order to form a spire on the summit. In Fig. 93 let A N D and B M C be the horizontal traces of the two faces of the tower. Begin by fixing the apexes S' and s' of extrados and intrados, taking care so to place them, relatively to each other, that the thickness of the conical vault diminish as the apex is approached. This is in order to lighten the weight of the vault without lessening its stability. It is evident that the apexes S' and s' should be situated upon the axis of the tower. Then proceed with the arrangement of the vault, employing for bed joints conical surfaces perpendicular to the intrados and with their respective apexes situated upon the axis of the cone. Form the vertical joints by planes passing through the axis of the cone, and arrange them to be continuous in alternate courses. In order to avoid the acute angle F'E'B' the construction K'G'H' may be adopted.

45. Cutting the Voussoirs:- First Method: Prepare a stone with six faces, two horizontal and four vertical. The two horizontal faces r y x u and s t v (Fig. 94) will have the form of the pattern h h"i"i (Fig. 93) on which is horizontally projected the voussoir in question, and will be distant from one another by an amount equal to m"m'. The vertical faces will be composed of two concentric cylinders u t v x, y r s, and of two rectangular planes r u t s, y x v. The stone will thus take the form of a stone in a vertical cylindrical wall.

Now apply upon the two vertical plane faces a pattern cut to the contour a'b'd'c', taking care that the four vertices are conveniently placed, that is so that b s = b'm', c t = c'n', d u = d'n", and a t = a'n'. Then by means of templets trace the arcs d d_1, a a_1, and with a flexible straight-edge applied successively upon each cylindrical face trace c c_1 and b b_1 (the latter is invisible upon the figure). Finally divide into the same number of equal parts the four arcs a a_1, c c_1, d d_1, b b_1, which will furnish points through which to pass a straight-edge in executing the four conical surfaces between which the stone is comprised.

The above method of cutting entails considerable loss of stone, to avoid which the following method has been applied:

46. Second Method:- In Fig. 95 let B D D"B" be the horizontal projection of one of the voussoirs composing a right conical vault. Suppose this voussoir cut, midway of its length, by a plane parallel to the vertical plane and containing the axis of the vault, so that the two parts of the stone as thus cut are vertically projected along the same lines. In the figure that portion in front of the secant plane is omitted in elevation, for the sake of clearness.

It is seen that the voussoir is comprised within a right prism having for a base the quadrilateral h'D'e'A' and the length of which is equal to the distance which separates, upon the horizontal plane, the two parallels v x and v"x", drawn through the points b and b", procured as below.

The horizontal projections of the curves in which the surfaces of intrados and extrados are cut by the two planes o e', o_1 A', perpendicular to the vertical plane, are the curves b A b", h G h", F e F", D c D", which determine the points b and b" through which the two parallels v x and v"x" are passed.

If now we revolve the two planes o e' and o_1 A' about x x" and v v" as axes, until they are parallel to the horizontal plane, we shall obtain in true size the two contours D D"f_1"f_1 and b_1 b_1"h_1"h_1, along which these two planes meet the different faces of the voussoir, prolonged.

Next describe from S' as a center, with radii S'E', S'A', two concentric arcs, and making them respectively equal to the horizontal arcs B A B" and F E F" there is obtained the pattern P P'R'R, which will be the development of the conical face of the extrados. Proceed in a similar manner to obtain the pattern M M'N'N, the development of the conical face of the intrados, taking s' for a center.

These constructions having been completed, begin by squaring a right prism (Fig. 96) of which the base a_2 h_2 d_2 e_2 is equal to the contour A'h'D'e' of Fig. 95, and of which the length e_2 e_3 is equal to v v"; then, upon the upper and lower faces trace the contours r s f_3 f_2 and m n b_3 b_2 by means of the patterns b_1 b_1"h_1"h_1, and D D"f_1"f_1 (Fig. 95), taking care that d_2 r = x D and h_2 m = z h. Next cut the two plane faces destined to form the vertical joints, being guided for each by the four right lines which limit it; after which dress the conical surfaces of intrados and extrados, being aided by a straight-edge, which always rests upon the curves limiting the faces, at reference points suitably chosen, as indicated upon the drawing. When these two surfaces have been cut, apply to them the two patterns of the development M M'N'N, P P'R'R, making them take the curvature of the surfaces, and trace the curves which are to serve as directrices of the second conical surfaces destined to form the bed joints, as shown in Fig. 97.

THE SPHERICAL DOME.

47. A spherical dome or vault is one whose intrados may be conceived as formed by a quarter-circle turning about one of its limiting radii, assumed to be in a vertical position. The simplest and the usual mode of arrangement of the voussoirs is in courses comprised, as regards

the intrados, between horizontal planes. Consequently the lines which, upon the intrados, separate the courses, are horizontal circles, projected horizontally in circles of the same size as the originals, and projected vertically in right lines parallel to the ground line. The lines which divide the courses into voussoirs are circular arcs resulting from the intersection of the vault with vertical planes passing through the center of the hemisphere. These lines are projected horizontally in right lines, and vertically in elliptical arcs.

In Fig. 98 let A'C'B' be the semi-circle resulting from the intersection of the intrados of the dome with a vertical plane parallel to the vertical plane of projection; let A D B be the horizontal projection of the circle forming the springing line of the dome (in that half of the horizontal projection which is above the diameter a b the spectator is assumed to be under the dome, while in the other half he is supposed to be above the dome, and the lines which are full in one half of the plan are thus broken in the other half); further, let a R b be the circle of the base of the vertical cylinder which forms the exterior face of the cylindrical wall upon which the dome is constructed.

In order to arrange the extrados of the dome describe a circular arc k'f'p' having its center upon the vertical f'O' and below the point O', so that the thickness at the key C'f' shall be consistent with the dimensions of the dome, the thickness increasing toward the springing line.

Divide the curve A'C'B' into such an odd number of equal parts as will give the desired number of courses. Then, through the points of division imagine horizontal planes to be passed cutting the dome in circles, which are horizontally projected in circles and vertically projected in parallels to the ground line, as shown in the figure. This system of lines will form the intradosal edges of the bed joints. In order to divide the courses into voussoirs, cut the dome by a series of vertical planes passing through the axis O O'C' and arranged equidistant for the sake of symmetry. These planes, A O, E O, G O, etc., will cut the sphere along great circles, which will be vertically projected in ellipses, such as G'H'I'C'. This curve is constructed by projecting upon the vertical plane the points G, H, I, where the trace G O of the corresponding secant plane meets the different circles which form the intradosal edges of the bed joints. These ellipses should be broken upon alternate courses, as indicated upon the drawing, in order more firmly to bind the voussoirs together.

The vertical joints as above determined will be normal to the surface of the intrados, and in order that the bed joints shall also be normal to that surface let them be formed by cones having their apexes at the center of the sphere, and having for their directrices the circles which have been adopted as the intradosal edges of the joints. These cones will cut the surface of the extrados, which is also spherical, along horizontal circles, which will be horizontally projected in circles equal to themselves and vertically projected in parallels to the ground line. Thus the cone which has for its directrix the circle K H P, K'P', will cut the surface of the extrados along a circle projected horizontally in k h p and vertically in k'p'.

If we consider a particular voussoir, that for example in the second course between the meridian planes O A and O G, we see that its horizontal

projection is in the contour k N I h and its vertical projection in the contour k'K'H'I'i'n'. The last course should be composed of a single stone which is to form the key, although this can be omitted, if it is desired to leave an opening, without compromising the stability of the dome.

48. Cutting the Voussoirs by the Method of Squaring:- Begin by squaring a right prism u v x y u'v'x'y' (Fig. 99) which has its two bases u v x y and u'v'x'y' equal to the pattern N I h k of the horizontal projection of the voussoir which it is proposed to cut, and of which the height x x' is equal to n'e'. Then, upon the lateral faces u u'y'y and v v'x'x of this prism trace the contours K_1 N_1 n_1 k_1 and H_1 I_1 i_1 h_1, employing for this purpose a pattern cut to the contour K'N'n'k' of Fig. 98 (The same pattern will answer for all the voussoirs of a single course, which voussoirs can indeed be considered as generated by the rotation of the pattern about the axis of the dome). In order properly to apply this pattern, care should have been taken previously to mark upon each of the faces two reference points N_1 and n_1, I_1 and i_1, by making N_1 u = I_1 v = I'm', and n_1 u' = i_1 v' = N n.

Next, upon the front cylindrical face, trace the circular arc N_1 I_1 by means of a flexible straight-edge applied upon the cylindrical surface; similarly upon the rear cylindrical face, with the same straight-edge, trace the circular arc k_1 h_1. Finally, upon the lower and upper faces trace the two arcs K_1 H_1 and n_1 i_1 identical with the arcs K H and n i, using for this purpose templets cut along these latter arcs.

The tracing being thus completed, begin by cutting the spherical intrados by moving upon the arcs N_1 I_1 and K_1 H_1 a templet cut to the curvature A'K'N'B'. Care is to be taken that in all its positions this templet correspond to a meridian plane, which can be assured if the arcs N_1 I_1 and K_1 H_1 be first divided into the same number of equal parts. The upper and lower joints, which are conical surfaces, will be cut by employing a straight-edge, to be moved upon the two arcs which limit each surface, these arcs having first been divided into the same number of equal parts. The extrados will be cut by aid of a concave templet cut along the arc k'n', but most often this surface will be left more or less rough.

The method which has been outlined is believed to be the most exact in its results, but it has the serious disadvantage of wasting a great deal of stone and a great deal of labor.

49. Second Method by Squaring:- This method is analogous to that previously described for the vertical conical vault. First project the voussoir which it is proposed to cut upon a meridian plane dividing it into two symmetrical parts, as shown in Fig. 100. Then cut a right prism having for its base a pattern of the vertical projection M'N'P' p'q'm' and a length equal to n n_1, after which lay off on the edges of this prism the lengths S m, U M, V N, and V N_1, X q, Y p, and Y p_1, Z P and Z P_1, and then mark upon the two bases the points n, n', and n_1, n_1', thus furnishing four points for each of the vertical planes n P and n_1 P_1, that is to say, four right lines which shall serve as guides in cutting the two plane faces. When these have been cut, trace the contour of the vertical joints by means of a pattern cut along the principal meridian section M'Q'q'm'. Finally, upon the horizontal faces q'p' and M'N' indicate the circular arcs p q p_1 and N M N_1.

One may then cut the upper conical joint, of which are known the two extreme elements P p, P'p' and P_1 p_1, P'p' and a directrix p q p_1, p'q', by employing a straight-edge, made to glide upon the directrix and always converging with the two extreme edges. The work should be verified by applying a flexible pattern cut to the development of the conical joint. When this pattern has been applied to the concave surface it will remain only to trace upon the surface the arc P Q P_1, P'Q', following the lower edge of the pattern. The lower conical joint will be executed in the same manner, using as directrix the arc n m n_1, m'n', already traced, and the corresponding pattern from the development, which pattern will permit of indicating the circular arc N M N_1, M'N'.

It then remains only to cut the spherical intrados, employing for this purpose a templet cut to the curvature of the principal section of the dome, and made to glide upon the two parallel arcs P Q P_1, Q'P', and N M N_1, M'N'. The templet should in each position pass through reference points fixed in advance by dividing each of the two arcs into the same number of equal parts.

This method of cutting, though avoiding certain disadvantages of the first, is less exact.

50. The "Bowl" Method:- If we join (Fig. 98) the point K to the point H and the point N to the point I, we shall obtain two right lines K H and N I which are evidently parallel and which are consequently in the same plane; further, they are the two bases of a trapezoid K H I N of which K N and H I are the other two sides, and of which K I is one of the diagonals. But this trapezoid is the projection of another trapezoid the four vertices of which are upon the intrados of the spherical dome.

It is necessary first to determine the true length of each of the sides of the trapezoid and of the diagonal which meets the vertices K, K', and I, I'. Now the side which unites the vertex N, N', to the vertex I, I', being horizontal, is evidently equal to its horizontal projection N I; and the same is true of the opposite side, which is equal to its horizontal projection K H. The other two sides are equal, and have for true length the distance of the point K' from the point N', that is to say, the chord of the arc K'N'. Finally, the diagonal has for its true length the hypothenuse I y of a right-angled triangle I y K, constructed upon I K as a base and with a height y K equal to m'I', that is to say, to the difference of height of the points K, K', and I, I', above the horizontal plane.

This granted, after having leveled one of the faces of the block of stone which has been chosen, construct upon it a trapezoid identical with that which has been mentioned, in the following manner: Trace, in the first place, a right line a b (Fig. 101) equal in length to K H; then, from a as a center, with a radius equal to I y, describe the arc u v; next, from b as a center, with a radius equal to the chord of the arc K'N', describe the second arc x z, which will meet the first at a point c; through this point c draw a parallel c d to a b, upon which take a length c d equal to N I; finally, join the point d to the point a. The trapezoid thus having been constructed, pass a circle through its four vertices a, b, c, d, of which the center o will be at the intersection of perpendiculars r o and s o erected at the middle points r and s of the sides d c and d a.

Assume this construction to have been completed upon one of the faces of a stone, as shown in Fig. 102. Then cut out the stone within the circumference N_1 I_1 H_1 K_1, giving the concavity the form of a sort of bowl, having the same radius as the spherical dome. To do this, employ a templet cut to the curvature of a principal section of the dome, taking care that the templet rest always upon the circumference N_1 I_1 H_1 K_1; that its plane be always maintained perpendicular to the plane of the circumference; and that in each of its positions it pass through two reference points, marked in advance, at the extremities of the same diameter, such as the points e and f, g and h.

When this bowl has been completely and accurately cut, trace within it the true contour of the intrados of the voussoir. To this end cut a templet to the curvature of the principal section of the dome, and chamfer the edge, if thick, for greater precision; apply it within the bowl in such a manner that it coincide perfectly with it, and that it rest exactly upon the points I_1 and H_1; and trace the arc I_1 H_1. Similarly, with the same templet trace the arc N_1 K_1. Finally, with other templets cut upon the parallels N I, K H, trace the arcs N_1 I_1, K_1 H_1.

The contour of the intrados being then perfectly indicated, cut the bed joints and vertical joints. For this purpose use a bevel with one branch curved and the other straight. The curved branch a b (Fig. 103) will be cut along a great circle of the sphere which forms the intrados of the dome, that is to say, along the curve A'K'C'B'. The straight branch will be fixed to the curved one in such a manner that its edge b c is perpendicular to a tangent to the curve a b at b. In using this bevel apply the curved branch in the cut bowl, taking care that the plane formed by the two branches be always perpendicular to the contour of the intrados.

Then cut away the stone along the four arcs which form the contour of the intrados, and form thus the four joint faces, which should coincide with the straight branch of the bevel, applied as above directed. The extrados, as previously stated, is usually left rough.

51. Method by Intradosal Patterns:- This method is applicable only when the radius of the dome is considerable, say 20 ft.; it is then very advantageous, being much simpler than the preceding method, and presenting more precision than the employment of templets for marking the outline of the intrados, which always leave some uncertainty. It is founded upon the principle that, in the case of a dome of large radius, the meridian arcs, such as K'N', which determine the height of the intrados of each course, differ but little from their chords; so that one can consider, without appreciable error, the intrados of each course as being a portion of a cone of revolution, which has its apex upon the axis of the dome, and which would be generated by the chord of the meridian arc corresponding to the course in question.

Now the cone is developable; consequently, if we prolong the chord P'S' to the point V', where it meets the axis of the spherical dome; if, further, from this point V' as a center, with V'P' and V'S' for radii, we describe the circular arcs P'z and S'x; and if, finally, upon these arcs we take lengths respectively equal to P v and S u, we shall obtain a pattern P'z x S' which will be the development of the intrados projected horizontally in P S u v.

The pattern of the intrados having been constructed as directed, apply it in the bowl (Fig. 102) cut as has been previously explained, making the pattern coincide with the concave surface, with its four vertices at the points N_1, I_1, H_1, K_1, marked in advance upon the circle e h f g which limits the bowl. At a single operation the contour of the intrados can then be traced, after which complete the cutting of the voussoir as directed under the preceding method.

GROINED AND CLOISTERED VAULTS.

52. The Groined Vault:- This term is applied to a vault the intrados of which is formed by the union of the soffits of two arches which meet, these arches having the same springing plane and the same rise (Fig. 104). The cylinders of the soffits of the two arches intersect along two curves or groins, which are plane curves, and which consequently are projected horizontally in right lines which are the diagonals of a parallelogram formed by the inner faces of the abutments, assuming the two arches to be circular or elliptical. It will be seen from the above, that the right sections of the two arches which form a groined arch are dependent, one upon the other, and that, one being given, the other must be consistent with it.

Let the two arches be as shown in Fig. 104. Upon A'B' as a diameter describe a semi-circle A'L'B', which will be the principal section of one of the arches, and draw the diagonals A C, B D, upon which will be projected the curves of the intersection of the two arches. Now divide the semi-circle described upon A'B' into a suitable odd number of equal parts, and through the points of division pass planes M'O'O, N'O'O, etc., passing through the axis O'O of the arch. These planes, which furnish the joints, will cut the intrados along right lines which will be the intradosal edges, and which will be horizontally projected in parallels to the axis O'O; but the continuity of these parallels will be interrupted in the angles A O D and B O C. Through the points such as m and n where these projections meet the diagonals draw, parallel to the axis O O" of the second arch, right lines such as m M_1 and n N_1, which will be the horizontal projections of the edges of the joints of this second arch. It will be noticed that these right lines also undergo a break of continuity in the angles A O B and C O D.

Take then a ground line A_1' D_1', perpendicular to the axis O O", above which lay off the ordinates u_1 M_1', v_1 N_1', x_1 L_1', etc., respectively equal to the ordinates u M', v N', x L_1', etc., of the principal section A'M'N'B'; and through the points A_1', M_1', N_1', L_1', etc., pass a curve A_1' M_1' N_1' L_1' D_1', which will be the principal section of the second arch and will here be a semi-ellipse. Then, through the same points, M_1', N_1', L_1', etc., draw right lines normal to the curve A_1' M_1' N_1' D_1', which will be the vertical traces of the joints of the second arch.

In order to construct these normals one may employ a property of tangent planes. For this purpose, at the point M' draw the tangent F'M' to the semi-circle A'M'B'; through the point F' where this tangent meets A'B' prolonged draw a parallel F'f to the axis O'O, which will meet at

f the diagonal C O A prolonged; through the point f pass another right line f F_1', parallel to the other axis O O", which will cut at F_1' the ground line D_1' A_1'; finally unite the two points F_1' M_1' by a right line, which will be the tangent at M_1' to the curve of the second arch. It will only remain then to draw through the point M_1' a right line M_1's perpendicular to this tangent and one will have the normal at the point M_1'. Proceed similarly for the other normals.

The vertical joints will be formed by planes perpendicular to the elements of each arch, care being taken to make them alternate in the courses, as indicated in the drawing. Then the polygon R N n N_1 R_1 z will be the horizontal projection of one of the groin stones, that is to say, of one of the voussoirs contiguous to the groin curve A O C and consequently forming at the same time a part of both arches. These are the only voussoirs that require special mention, since the others, not participating at the same time in both arches, will be cut like those of any simple arch.

To form the extrados of the vault, proceed as follows:
After having determined as usual the right section G'R'Q'P'H'I' of the extrados of the first arch, construct from it the right section of the extrados of the second arch just as was done for the intrados, that is to say, for each point such as P_1' take an ordinate y_1 P_1' equal to the corresponding ordinate y P' of the first arch.

In order to determine the horizontal projections of the edges along which the bed-joint planes meet the extrados, let fall from the extremities of the joints perpendiculars P_1' y_1 and P'y upon A_1' D_1' and upon A'B', and draw through the points y_1 and y right lines y_1 p and y p, respectively parallel to O"O and O'O. These right lines will be the projections in question and will meet at a point p, which joined to the point n will furnish a right line p n, which will be the projection of the right line along which the joints N'P' and N_1' P_1' intersect.

If it is desired to know the true shape of the two identical groin curves A O C and B O D, through the points U, V, X, Y, etc., erect perpendiculars U U_2, V V_2, X X_2, etc., respectively equal to u M', v N', x L', etc., and through the points U_2, V_2, X_2, pass a curve B U_2 V_2 D, which will be the curve common to the two groins.

53. Cutting the Voussoirs:- Begin by squaring a right prism of which the base a b c d e f (Fig. 105) is identical with the horizontal projection R N n N_1 R_1 z of the voussoir which it is desired to cut, and of which the height is equal to K P_1'. Upon the front face mark the contour S"T"M"N"P"Q"R" of the head of the voussoir by means of a pattern cut to the contour S'T'M'N'P'Q'R' (Fig. 104). Similarly, upon the lateral face e e'd'd trace the contour S"'T"'M"'N"'P"'Q"'R"' by means of a pattern cut upon S_1' T_1' M_1' N_1' P_1' Q_1' R_1', after which draw the horizontals M"m" and m"M"', S"z' and z'S"', R"z and z R"', N"n" and n"N"',etc.

Then cut the cylindrical intrados which passes through the arc M"N" by using a templet cut to that arc, which is to be moved along the two right lines M"m" and N"n", making it pass through reference points marked upon each of those lines. Cut in the same manner the intrados of the elliptical arch, which passes through the arc M"' N"' and meets the intrados previously cut along a curve m"n" which will be a portion of the groin. The other faces, which are plane surfaces, will be easily cut.

It has here been assumed that the arches meet at right angles; but this meeting may be at any angle whatever, as seen in Fig. 106, where the cylinders of the intrados alone are represented, without the mode of procedure being thereby changed. Only, the two groin curves, instead of being identical, will be unlike; and the edges of the intrados, instead of being perpendicular to each other, will make an angle equal to that at which the two arches themselves intersect.

54. The Vault with Double Groin:- illustrated in Figs. 107 and 108, but which will not be described in detail, is a vault whose groins are intersected by horizontal cylinders, springing from the corners of the abutments and united at the top of the vault by a flat vault having the form of a parallelogram.

55. The Cloistered Vault:- This, like the groined vault, results from the intersection of two cylinders, having their axes in the same horizontal springing plane and having the same rise; only it differs in this, that the parts of the elements which are preserved in one are precisely those which are suppressed in the other.

Thus (Fig. 109) A B C D being a rectangular space which is to be vaulted, one adopts for the intrados a surface formed of four parts, belonging two and two to the same cylinder; the parts projected horizontally upon the triangles A O D and B O C will form part of the same cylinder having for right section the semi-circle A'G'B'; the other two parts, projected upon A O B and C O D, will belong to another cylinder, of which the right section will be the curve B_1' G_1' C_1', the point G_1' being at the same height above the springing plane as the point G'. It results from this new combination that the two groins projected upon A C and B D, instead of being salient, as in the groined arch, are re-entrant.

The general mode of arranging the voussoirs in the cloistered vault does not differ from that in the groined vault. The bed joints are alike furnished by planes passing through elements of the cylinders, and the vertical joints by planes perpendicular to those elements.

The cutting of the voussoirs at the angle, the only ones which present special difficulty, is effected in the same manner for both vaults. As in the groined vault, so in the cloistered, only one of the two arch sections can be arbitrarily chosen, and the other must conform to it. The key, and even one or more of the adjacent courses, can be omitted, if desired, without compromising the stability of the vault.

In architecture it is sometimes found desirable to flatten the upper part of the vault, as illustrated in Fig. 110.

56. Lunettes are vaults formed by the penetration of one arch into another, or indeed into any vault whatever, having the same springing plane but a different rise. In this respect they resemble constructions which have already been studied in which one arch penetrates another. They differ in that, in the Lunette, the courses of the penetrating arch must join those of the arch which it enters, which is supposed to be in this case, not of brick masonry, but of cut stone.

57. Right Lunette in an Arch:- This lunette is formed by the meeting of two arches which intersect at a right angle, having the same springing plane, but a different rise. In Fig. 111 let A'M'B' be a right section of the intrados of the smaller arch, and A"M"N" of the large arch, the latter section being here revolved into the vertical plane. The axis of the small arch is the right line O'O; that of the large arch is the right line o O_1.

Begin by dividing the principal section of the large arch into an odd number of equal parts, and proceed similarly for the small arch, taking care, however, that the first point of division L' be situated a little lower than the corresponding point L", for reasons which will shortly be explained. Then determine, as ordinarily, for both arches the bed joints and the extradosal surfaces.

The horizontal projection of the line of intersection of the two arches will be determined by cutting the arches by horizontal planes. Thus, if it is desired to find the point M of this projection, drop from M' upon the ground line the perpendicular M'M; similarly, from the corresponding point m" of the large arch drop upon the ground line a perpendicular, revolving the foot to m_1 by means of a circular arc; then through the point m_1 draw a horizontal m_1 M, which will meet the first perpendicular M'M at a point M, which will evidently be a point of the projection in question. Thus one will obtain as many points of the projection as are thought necessary.

When this projection A L M B has been determined, find the lines along which the joints of the small arch cut the intrados of the large arch. Now if the joint M'Q', in particular, be considered it will be seen that it cuts the intrados of the large arch along a curve M m, which will stop at the point m, where it meets the intradosal edge m S, which corresponds to the point M" of the right section of the large arch. This curve will be a portion of an ellipse, which, if it be prolonged beyond the point M, ought to pass through the point O, which will be its summit, and consequently to be tangent at that point to the springing line A B.

From the point m, the joint M'Q' cuts the joint M"Q" of the large arch along a right line m q, of which the extremity q is obtained, as shown upon the drawing, by means of a horizontal plane Q"q' passing through the point Q". It will be noticed that this right line m q ought to pass through the point o, where the axes of the two arches meet, since here these axes are the traces of the two joint planes, the arches being supposed full-centered.

Through the point q draw, parallel to the axis of the large arch, a right line q T, which will be the extradosal edge of the joint M"Q" of the large arch, which joint will have for its horizontal projection m q T V. On the other hand, the joint M'Q' of the small arch will cut the extrados of the large arch along a curve q Q, each point of which will be obtained by means of horizontal secant planes, as indicated on the drawing, and this joint M'Q' will have for horizontal projection the contour M m q Q e f. The curve Q P z will be that along which the extradosal surfaces of the two arches will meet.

It is easy to see why the first point of division L' of the right section of the small arch ought to be lower than the corresponding point L" of the large arch. For if it were otherwise, there would be joints of the small arch which would cut the intrados of the large arch, and vice versa, uselessly complicating the cutting of the voussoirs, and producing lines which would be disagreeable to the eye.

It will be noticed that the line projected in A M B (Fig. 111) is only a kind of groin curve, and the cutting of the voussoirs is so closely analogous to the corresponding operations for the groined arch as not to require special explanation. In Fig. 112 is given a view of the voussoir which has for its face, in the small arch, the contour M'Q'P'R'; and in Fig. 113 is given a view of the voussoir forming the key.

It frequently happens that the lunette alone is of cut stone, and that the two arches are of brick masonry, except the voussoirs at the groins, as seen in Fig. 114. The drawing is constructed in a manner similar to that already explained, but care is to be taken that the joints of the stone voussoirs coincide upon the arches with one of the bed joints of the courses of bricks which compose the masonry of those arches.

58. Skew Lunette in an Arch:- What has been said regarding the right lunette applies also to the skew lunette. The drawing, and the cutting of the groin stones, are executed in the same general manner. The only difference between the two cases is that the angle between the axes of the arches is right in one case and acute in the other. There is an objection to the skew lunette in that the angles which the intradosal surfaces of the voussoirs make with each other are acute, at least for the voussoirs situated on the side where the abutments of the two arches meet at an acute angle. To avoid this difficulty, the small arch A (Fig. 115) may be stopped at a vertical plane m n, and at this plane an elbow constructed so as to give the arch a direction perpendicular to that of the large arch B, thus replacing the skew lunette by a right lunette.

59. Skew Lunette in a Spherical Dome:- In Fig. 116 let A o_2 B be the springing circle of the spherical dome, and a_1 A and b_1 B the two springing lines of the arch which penetrates the spherical dome, all these springing lines being situated in the same plane, which will be adopted as the horizontal plane of projection. The vertical plane of projection will be taken as a plane passing through the center O of the spherical dome, and parallel to the axis o_1 o_2 of the arch.

This vertical plane will cut the spherical dome along a meridian section A"M"N", which will be divided into equal parts A"L", L"M", M"N", etc. Upon this same vertical plane revolve the right section of the arch, along A'L'M'N', and divide it into equal parts A'L', L'M', M'N', etc., so that the first point of division L' is lower than the corresponding point L" upon the spherical dome, for reasons explained in Art. 57. Finally, arrange the extradosal surfaces of the two vaults as ordinarily, and indicate the joints of each of them. The lunette is skew, since the axis o_1 o_2 of the arch does not pass through the center O of the spherical dome.

Now determine the horizontal projection of the curve along which the arch cuts the dome. To this end draw horizontal secant planes, such as M'm". This latter will cut the intrados of the arch along two intradosal edges, one of which, m_1 M, will meet at M the circle along which the intrados of the spherical dome is cut by the same horizontal plane M'm". This point M will be one point of the intersection in question, and in a similar manner can be obtained as many other points as thought necessary.

When the curve A M N B, has been determined, proceed to find the lines along which the joints of the arch cut the intrados of the spherical dome. Now, if the joint N'P' be considered in particular, it will be seen that it cuts that intrados along a circular arc, projected in the elliptical arc N Q. The point Q is obtained, as above, by means of a horizontal secant plane Q'N". If the curve be required with greater precision, intermediate points can be obtained by the same construction. Further, it will be seen that the curve should pass through the point o_2, and should be tangent at this point to the springing circle of the spherical dome.

The same joint N'P' cuts the conical joint of the spherical dome which is formed by the rotation of the normal O N" about the vertical axis of the dome, along a curve Q S. In order to obtain points of this curve, such as S for example, employ a horizontal plane S'P", which will cut the joint of the arch along the right line s_1 S, and the conical joint along a circle f S having its center at O; that right line and this circle will meet at a point S which will be the point in question. Other points can be obtained in the same manner.

After having cut the conical joint, the joint N'P' cuts the extrados of the spherical dome along a curve S R, the different points of which will be obtained by means of horizontal planes. It afterward meets the horizontal face R"T" along the right line R V, then the cylindrical face g V along a curve projected in V Q, and finally the extrados of the arch along the right line Q q_1. All these lines are determined by means of horizontal secant planes. By analogous constructions one will determine the different lines along which the other joints of the arch meet the different parts of the spherical dome.

For the sake of greater clearness, there are shown on the vertical plane the projection A_2 L_2 N_2 B_2 of the face curve of the arch upon the spherical dome, as well as of the intradosal edges of the different joints, although these projections are not necessary to the preparing of the stones.

60. <u>Cutting the Voussoirs</u>:- Take for example the voussoir which has for its face in the arch the contour M'N'P'Z' and for its horizontal

projection n_1 N Q D E F G. Begin by squaring a right prism (Fig. 117) having this projection for a base, and whose height is equal to the difference of level of the points m" and P" (Fig. 116). Then, upon the front face of this prism trace the contour M_1 N_1 P_1 Z_1 by means of a pattern cut to the contour M'N'P'Z'; mark in the same way upon the lateral face the contour E_1 E_2 J_1 K_1 D_1 C_1 H_1 by means of another pattern cut to the contour h T"R"P"N"M"k. Then, pass through the right line E_1 H_1 a plane face E_1 H_1 I_1 U_1, perpendicular to the face E_1 H_1 D_1 J_1 and such that one can apply to it its appropriate pattern. Afterward, cut the cylindrical intrados which passes through M_1 N_1, as well as the two joints which are contiguous to it, and which pass through the right lines P_1 N_1 and M_1 Z_1, which can be accomplished by means of a square, one branch of which is to be applied upon the face M_1 N_1 P_1 Z_1, to which that intrados and these joints are perpendicular. When it is judged that these faces have been sufficiently prolonged, apply to them their respective patterns.

The conical joint H_1 C_1 W_1 I_1 will be cut by means of a bevel formed by two rectilinear branches containing between them the angle h k M" (Fig. 116): while one of the branches glides upon the plane H_1 I_1 U_1 E_1, always resting upon it normal to the curve H_1 I_1, the other branch will describe the conical joint, upon which it will be necessary afterward to trace the circular arc C_1 W_1, which can be done by laying off upon this conical face, along several elements, a length equal to M"k (Fig. 116).

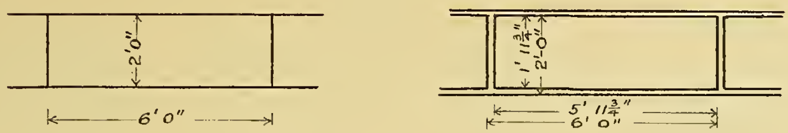
The spherical intrados will be cut as directed in treating of the spherical dome. Finally, the upper conical joint, passing through D_1 K_1, will be executed by means of a bevel, one branch of which will be curved and will form with the other an angle equal to M"N"P" (Fig. 116), the curved branch having a curvature equal to that of a meridian section of the spherical dome.

In Fig. 118 is represented the voussoir of Fig. 117 seen in its natural position.

61. Descents:- Descents, like lunettes, may give rise to a junction of voussoirs more or less complex, according as they are right or skew with reference to the arch or vault which they penetrate. What has been said regarding lunettes should serve to make clear the proper treatment of descents. It will often be preferred, however, to avoid this construction and to terminate the descent by a short horizontal arch A (Fig. 84) in order to prevent sliding of the voussoirs and the bringing of pressure upon the vault penetrated, and the problem will thus be reduced to that of the lunette.

62. Remark:- Sundry other problems than those treated in the preceding pages, and more or less common in architecture, will be found in the complete work, Traité Pratique de la Coupe Des Pierres, par Emile Lejeune.

Fig. 1



Stone as shown in drawing. Stone as cut and laid.

Fig. 2

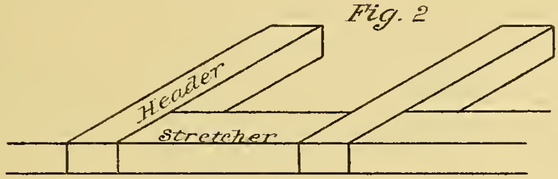
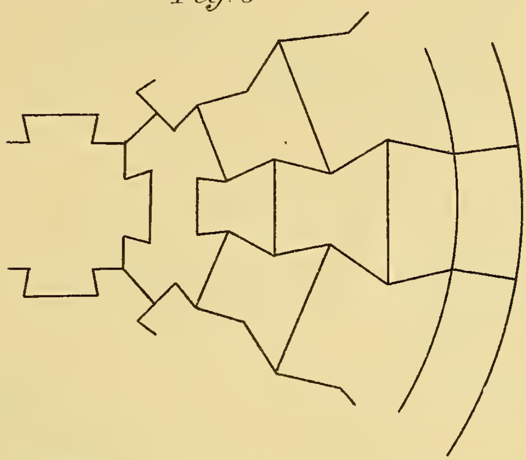


Fig. 3



Interlocking of stones.
Eddystone Lighthouse.

Fig. 4

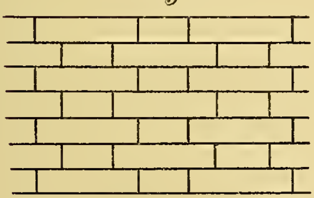


Fig. 5

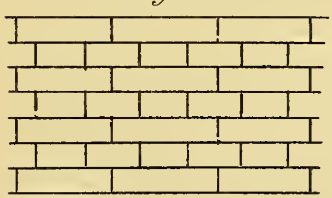


Fig. 6

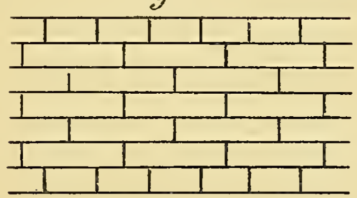


Fig. 7

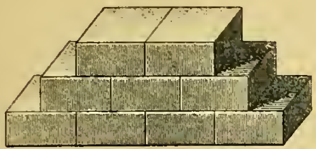


Fig. 8

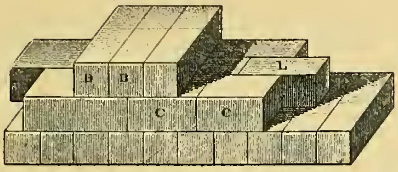


Fig. 9

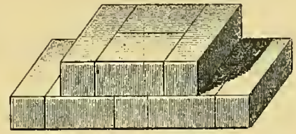


Fig. 10

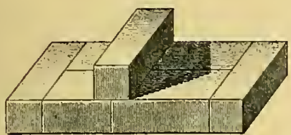


Fig. 11

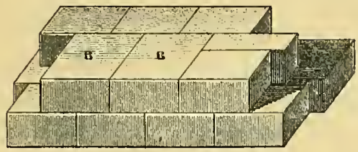


Fig. 12

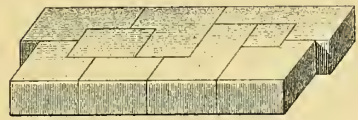


Fig. 13

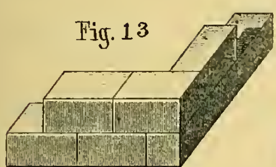


Fig. 14

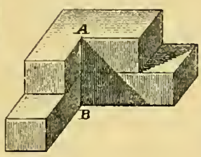


Fig. 15

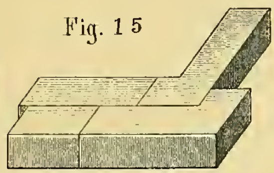


Fig. 16

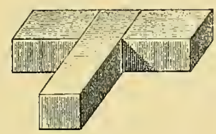


Fig. 17

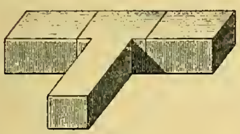


Fig. 18

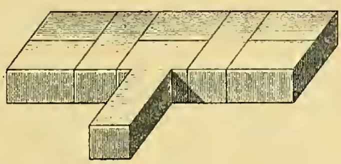


Fig. 19

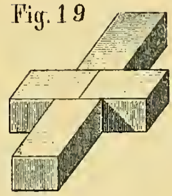


Fig. 20

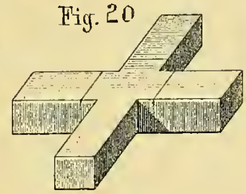
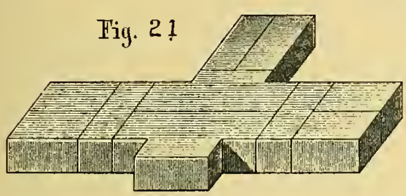


Fig. 21



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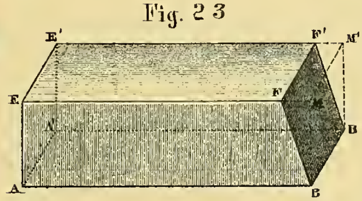

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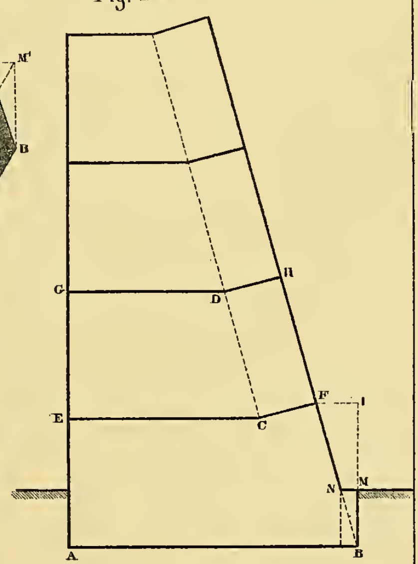

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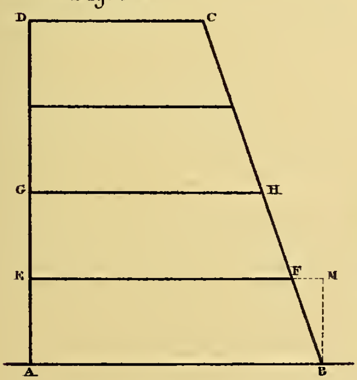

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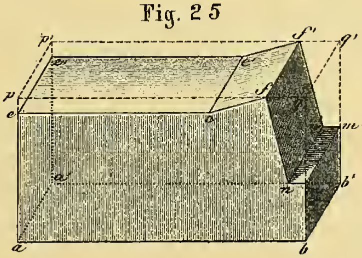

Em. Lejeune del.

Fig. 26

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Workman.

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Quality of Work Required.

Good cut to polish

Sketch

Color ____________

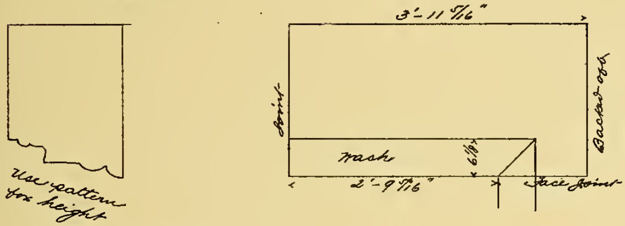

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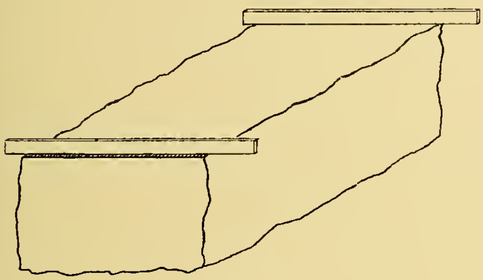
Fig. 27



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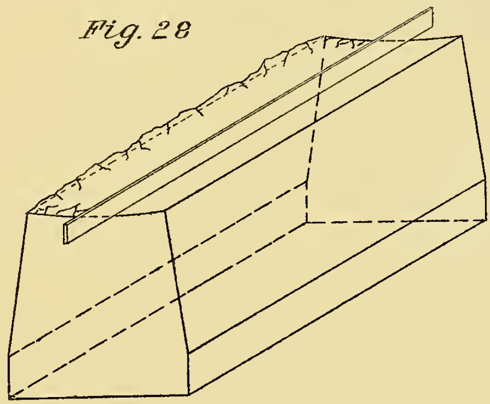
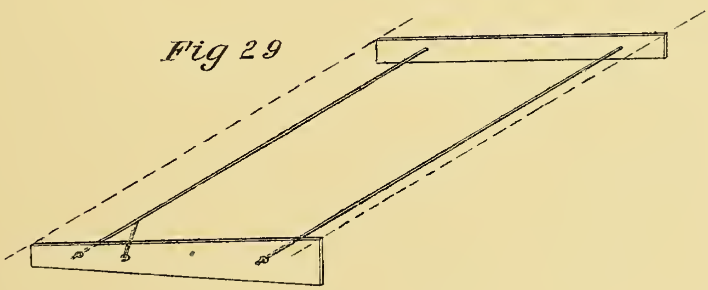


Fig 29



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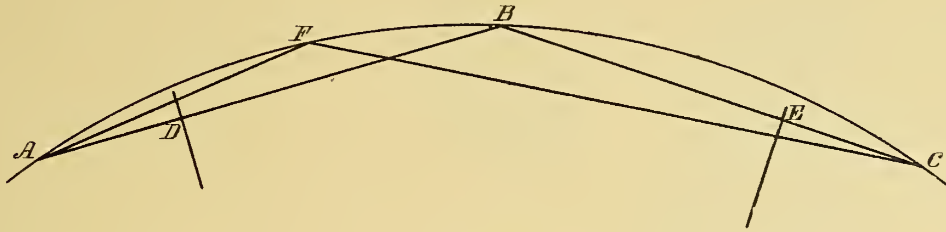

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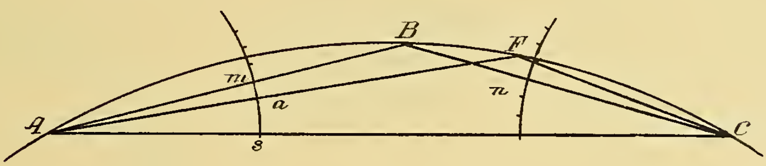

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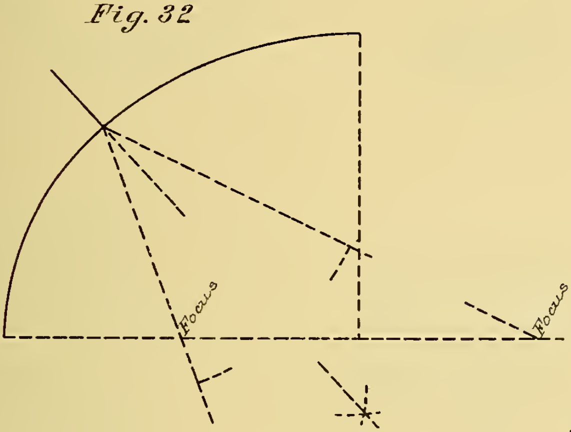

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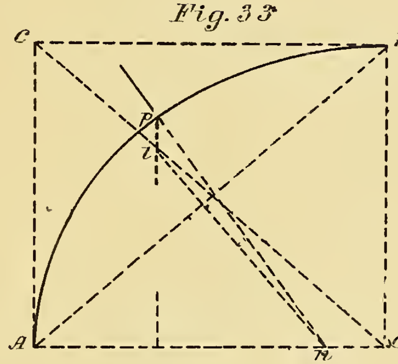

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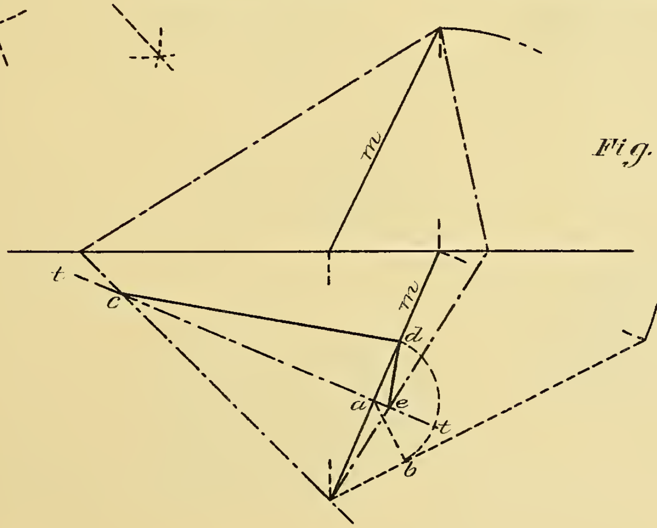

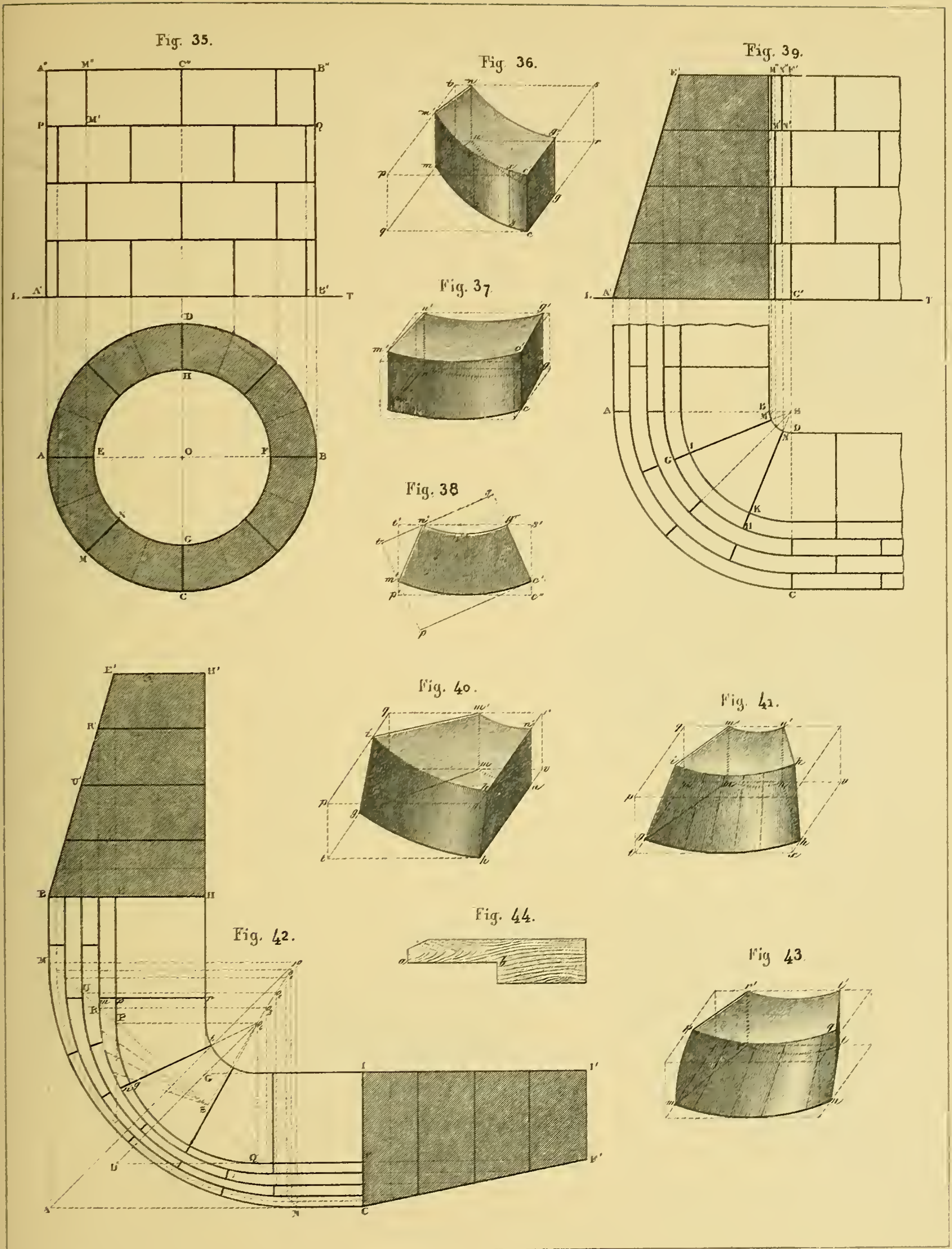

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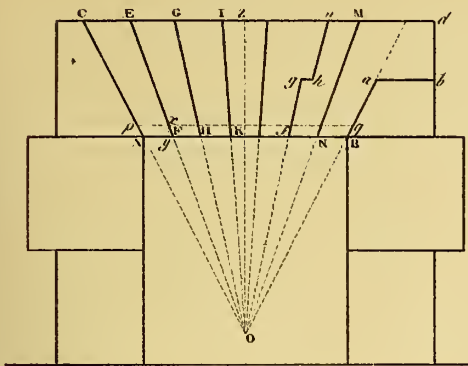

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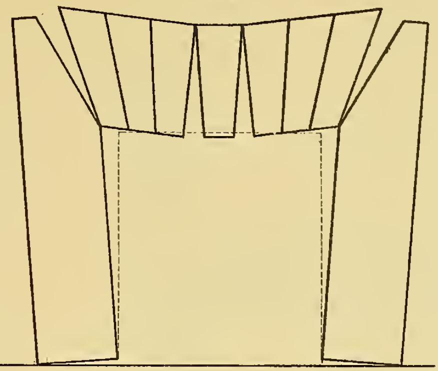

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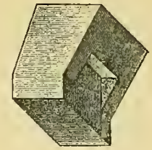

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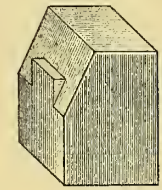

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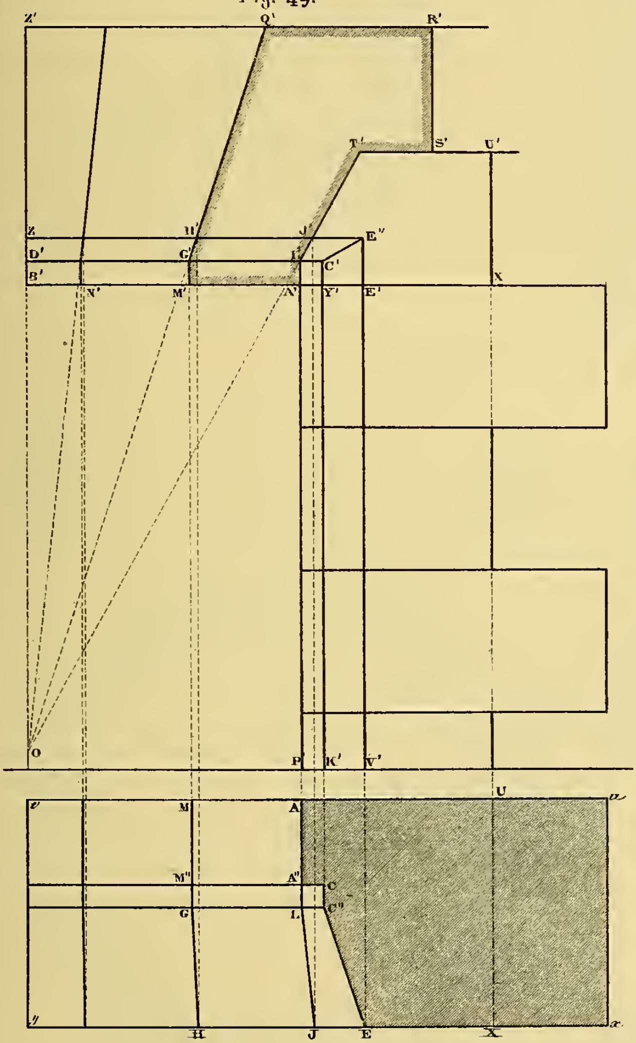

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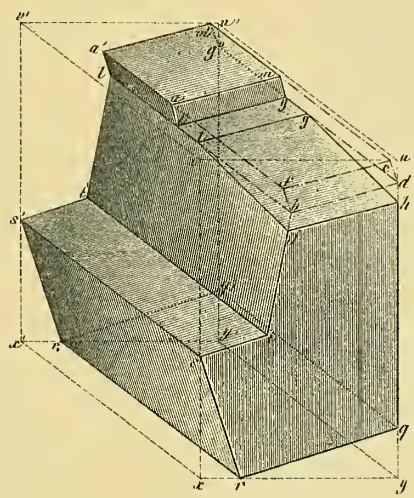

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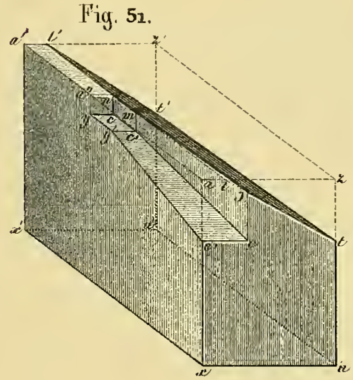

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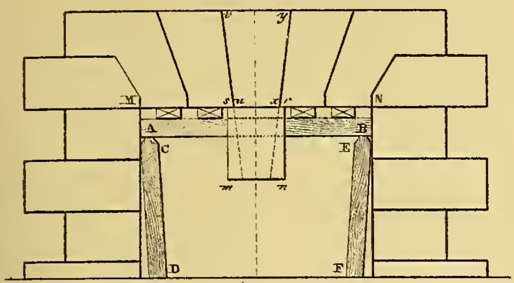

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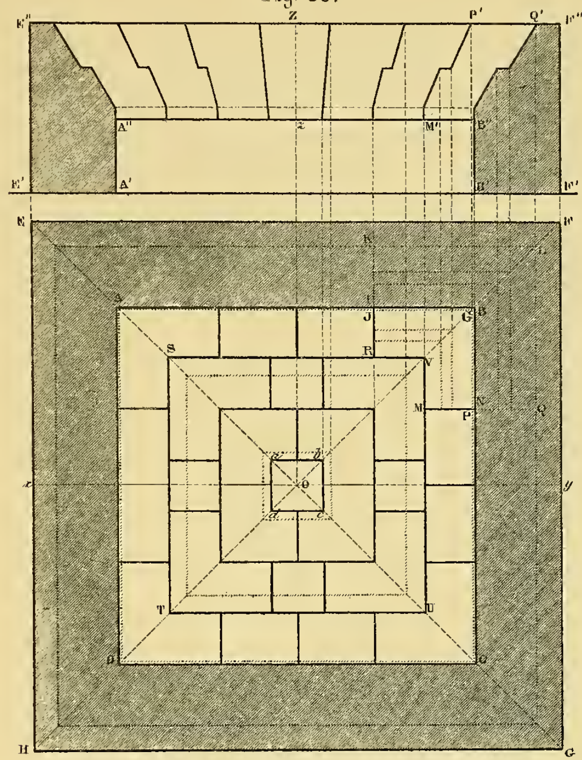

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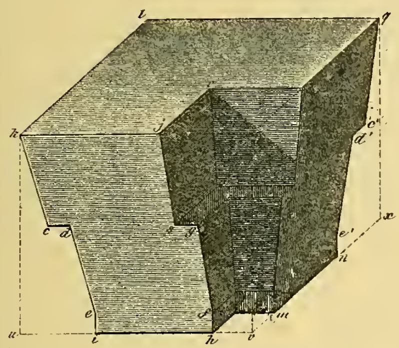

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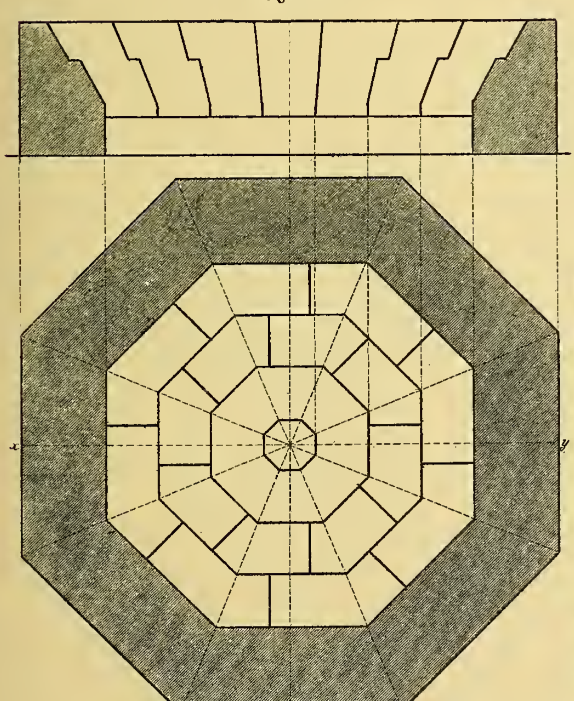

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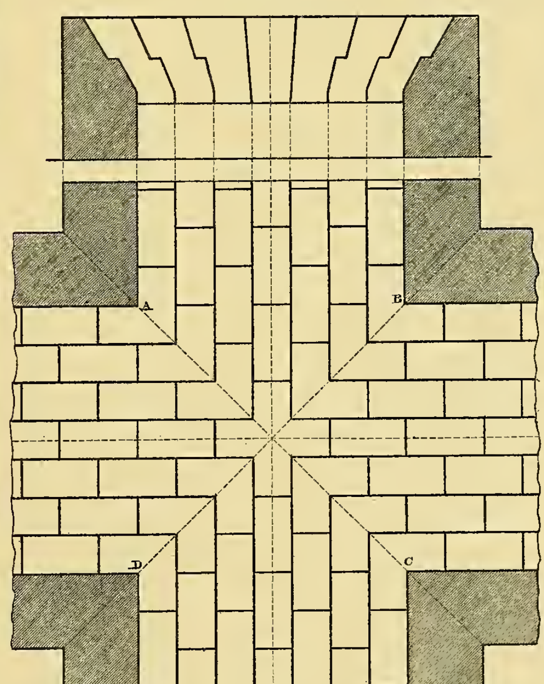

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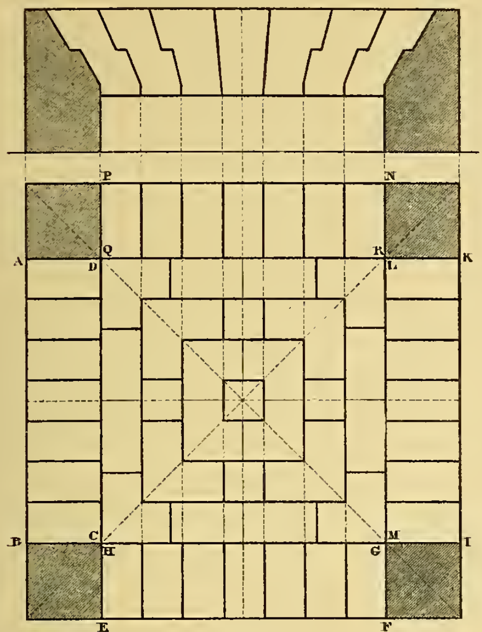
Fig. 57



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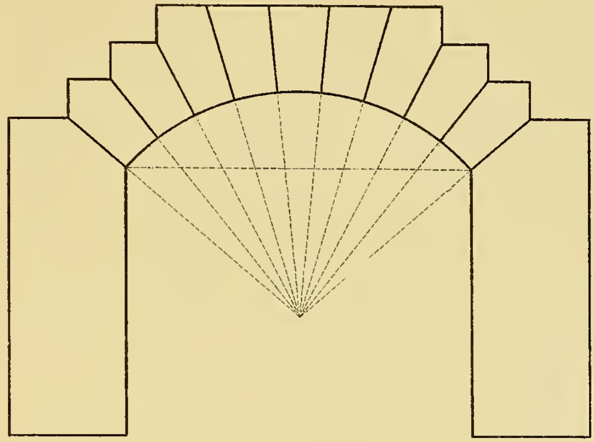

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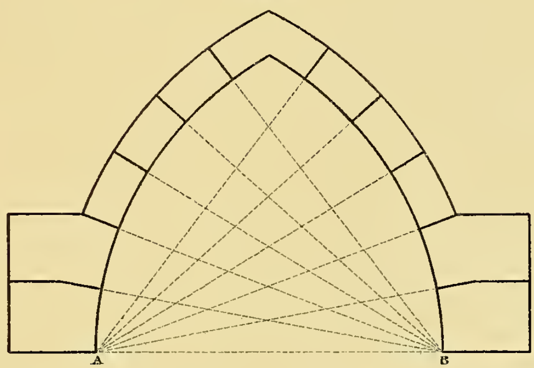

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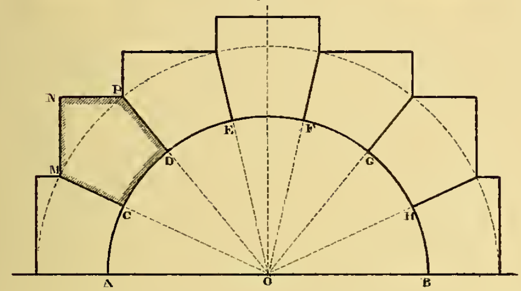

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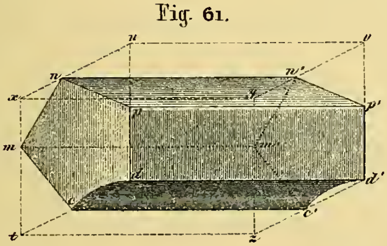

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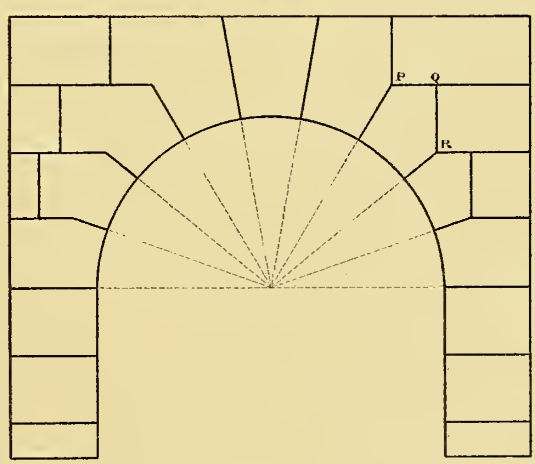

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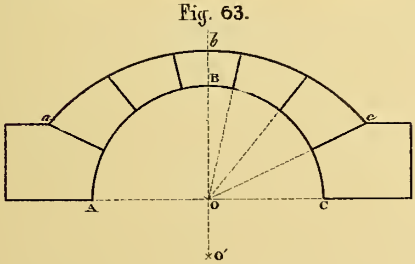

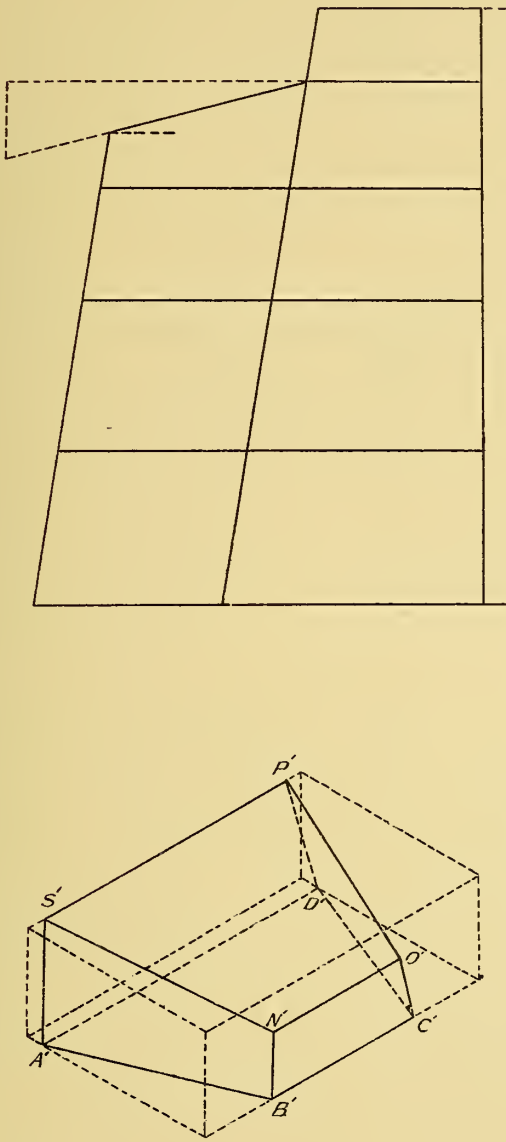

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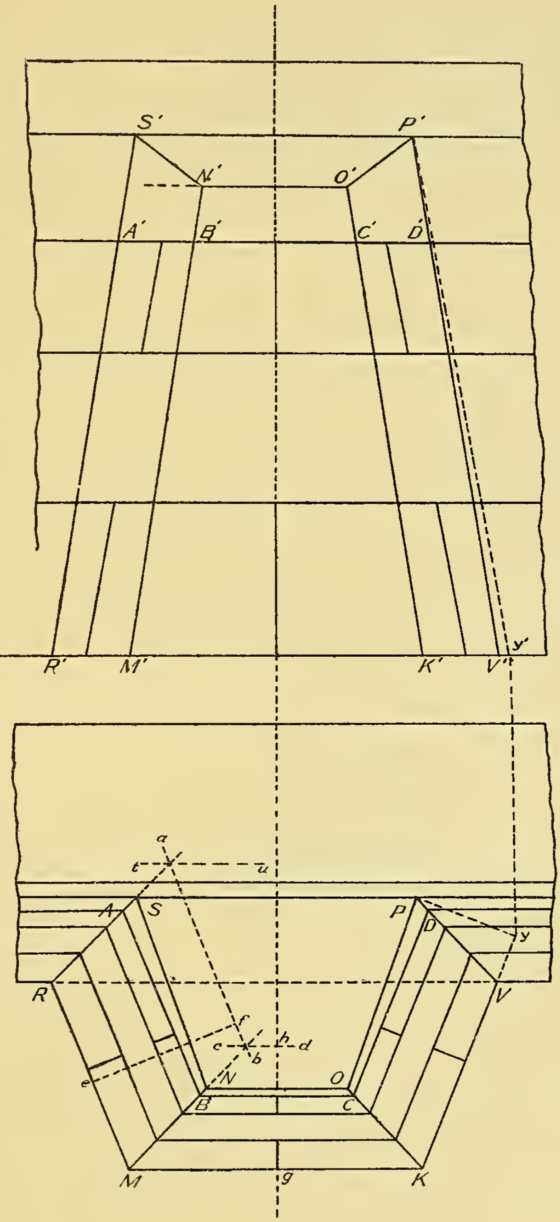

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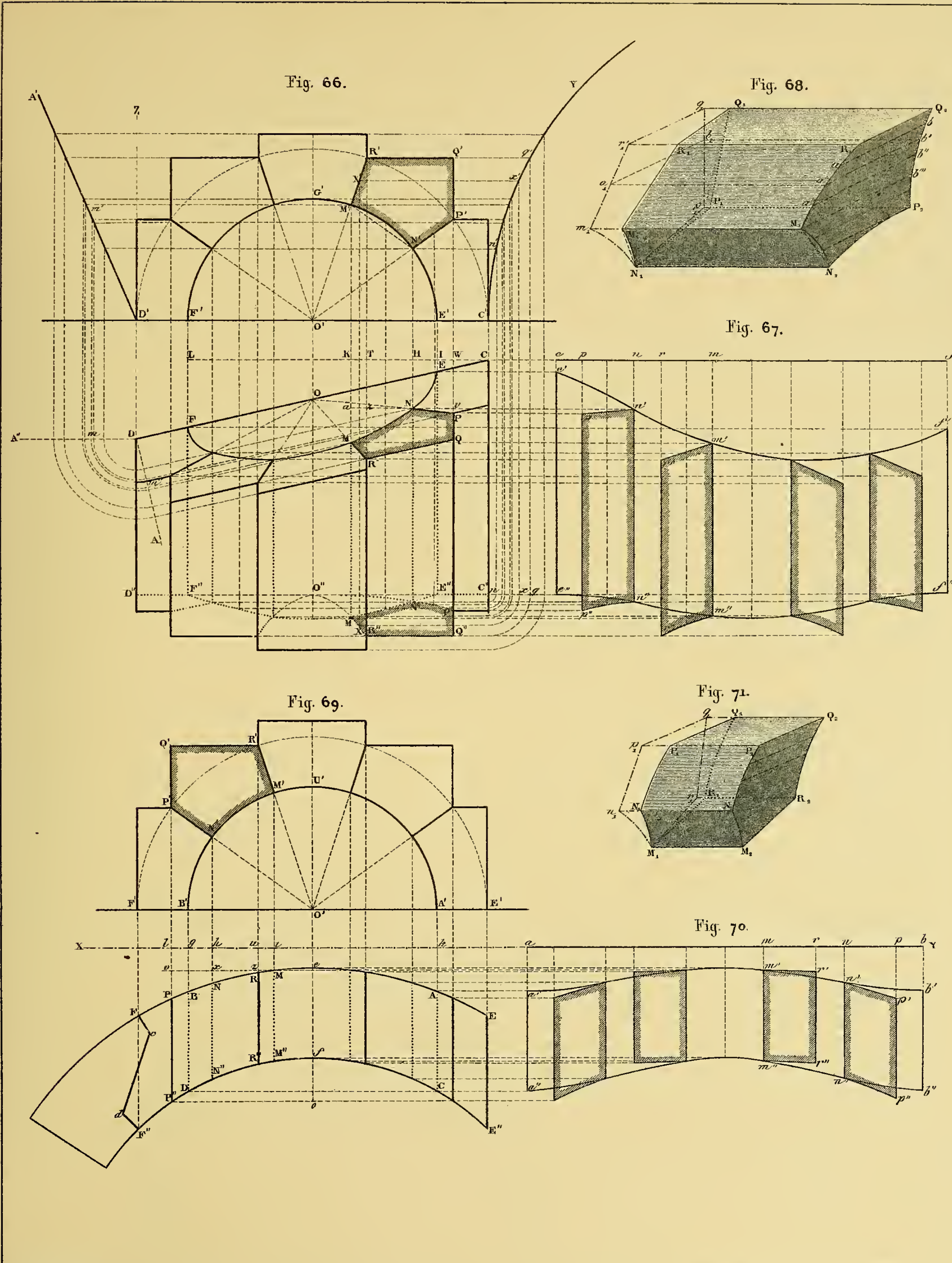

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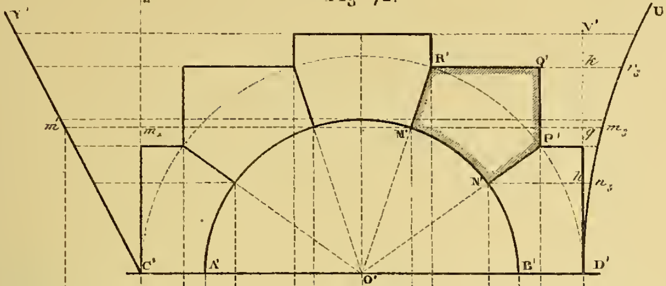
Em. Lejeune del.

Fig. 72.



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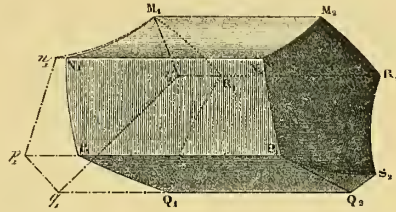

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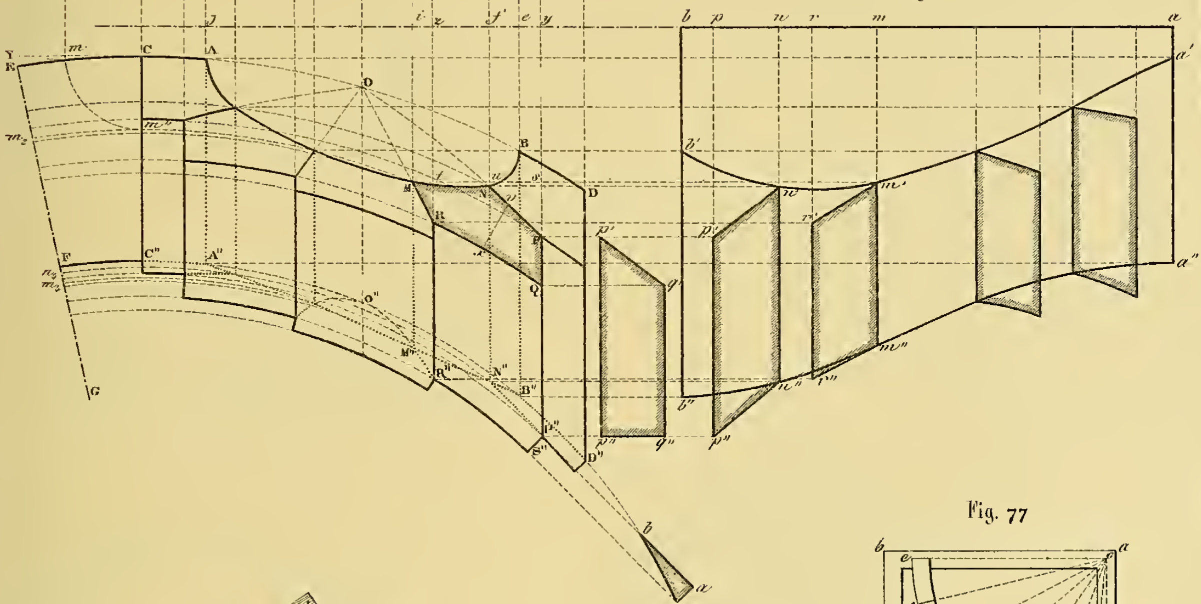

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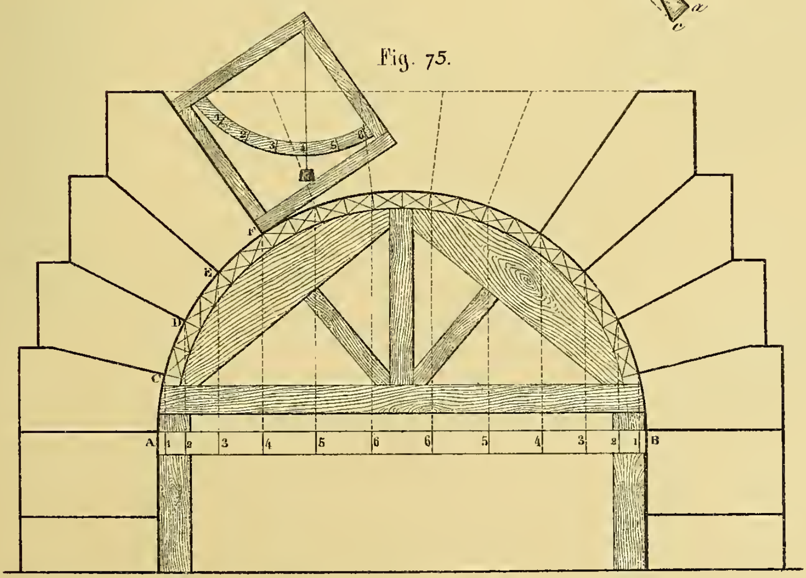

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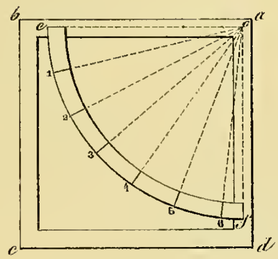

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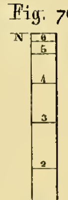

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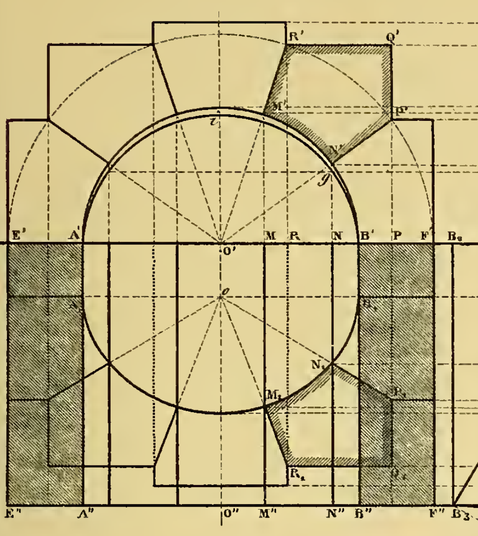

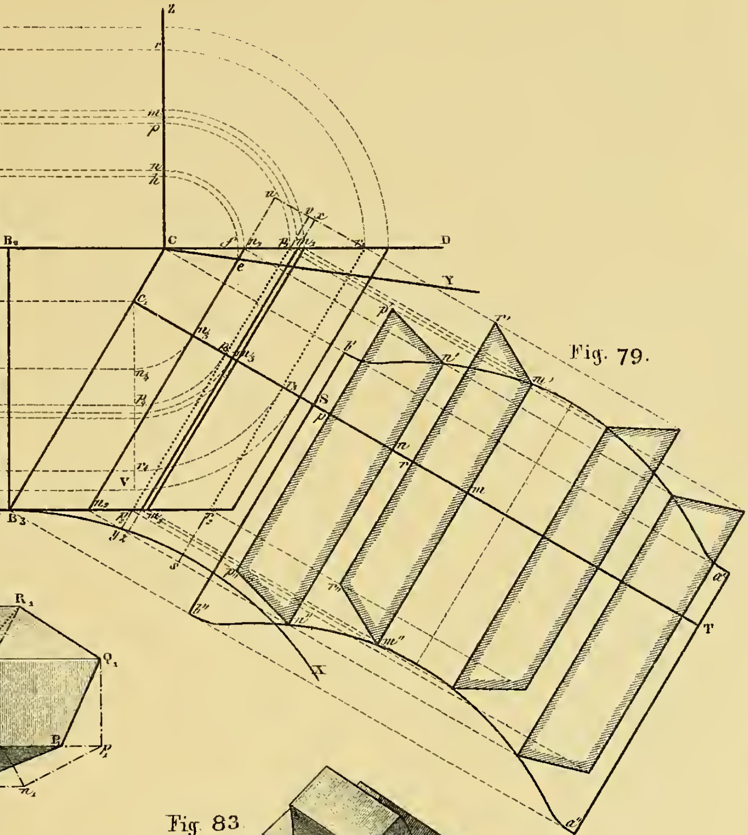

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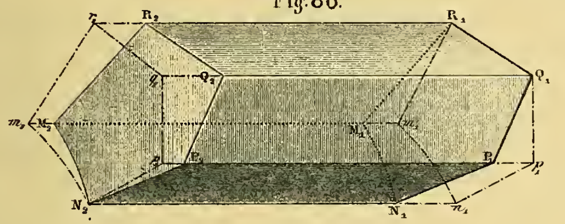

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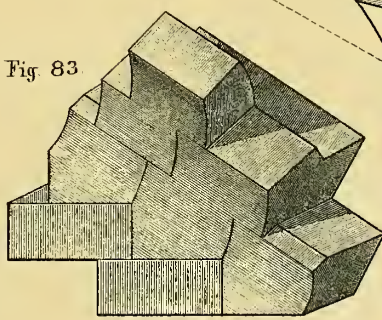

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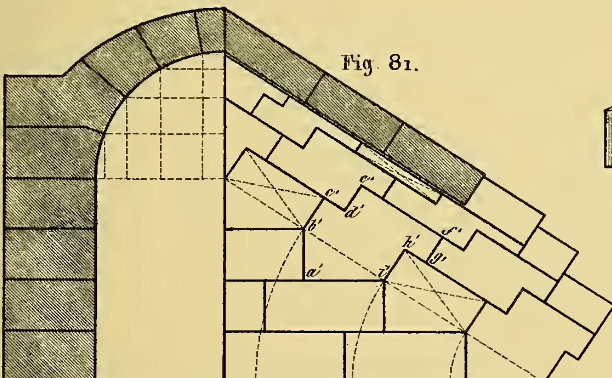

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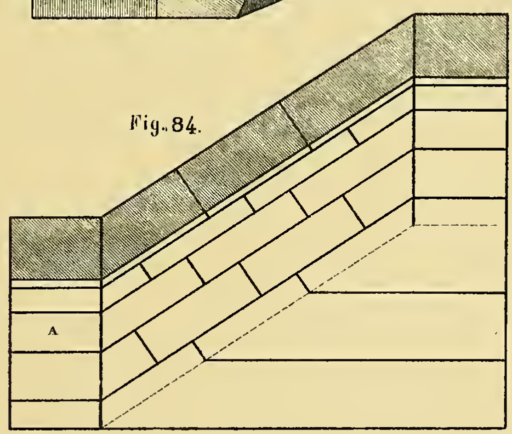

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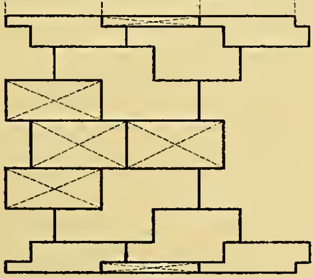

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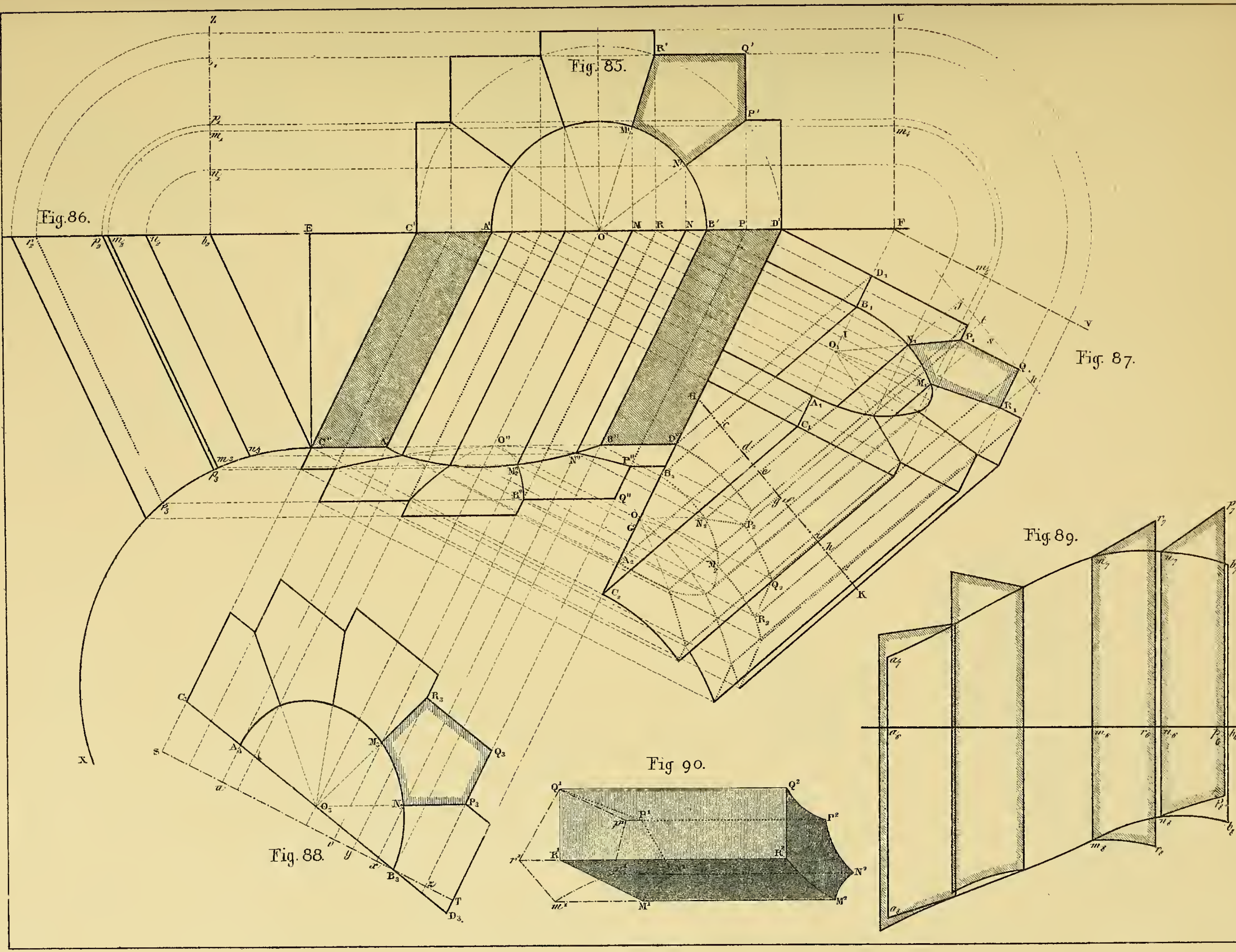

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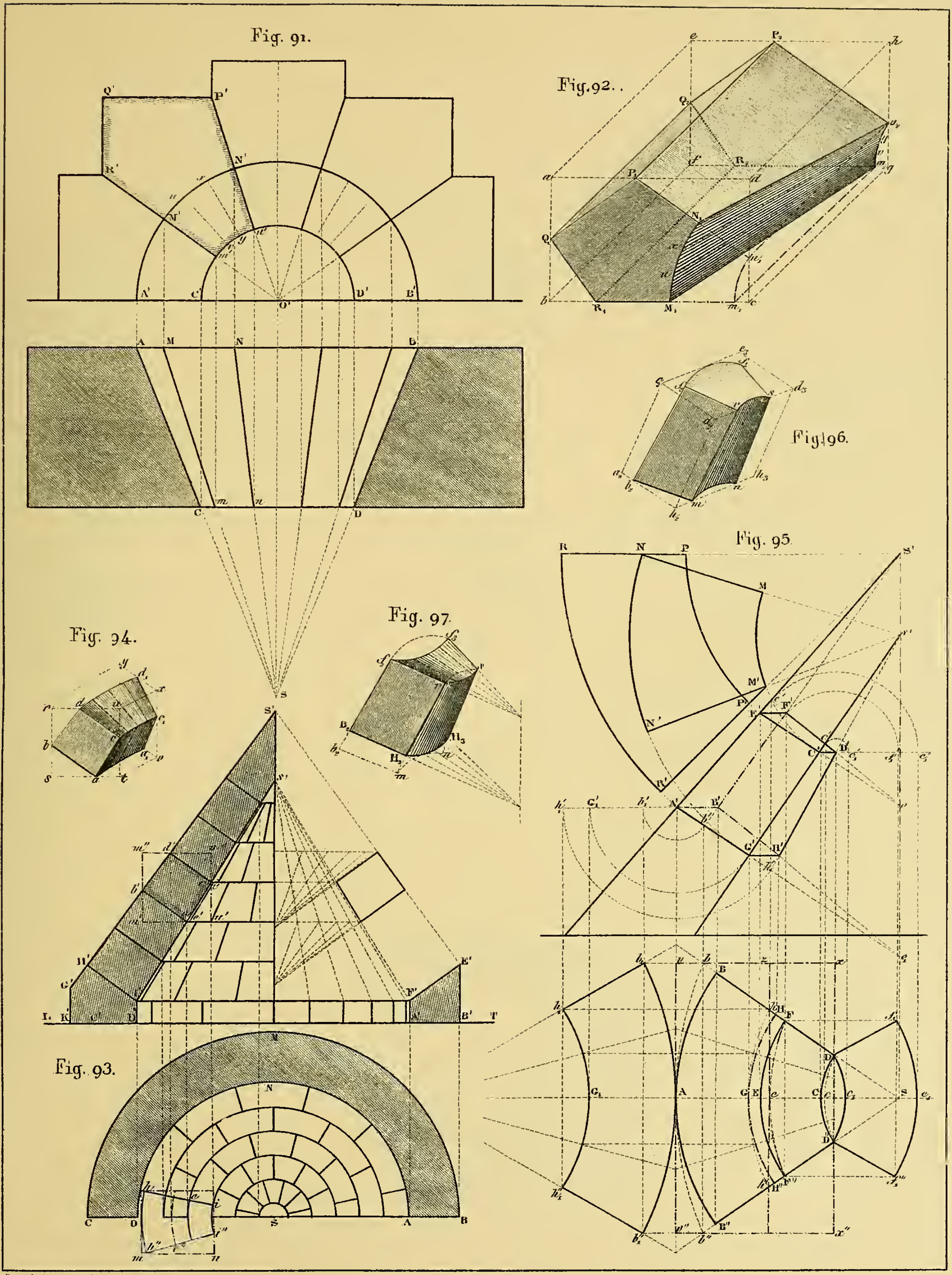
Em Lejeune del.



Em. Lejeune del.

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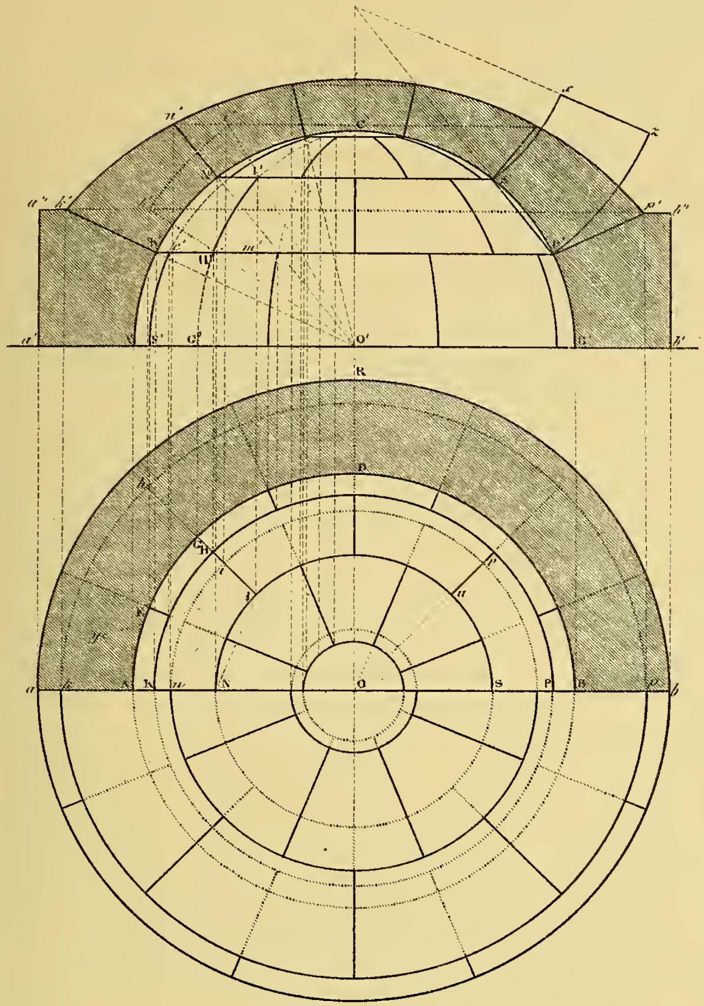

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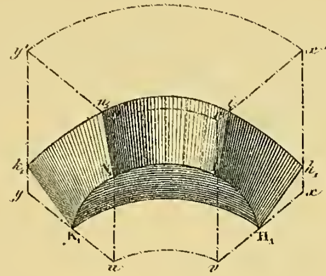

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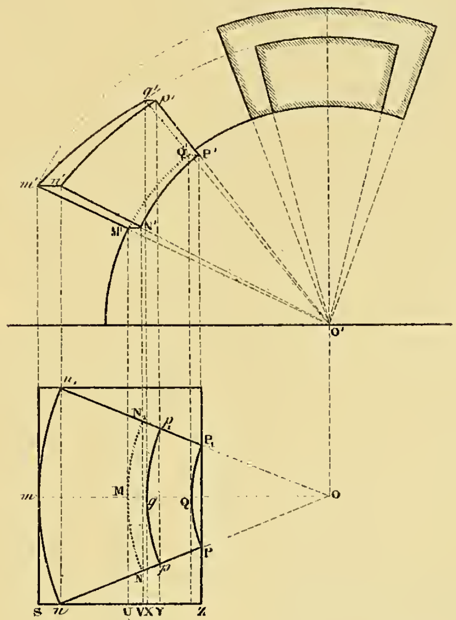

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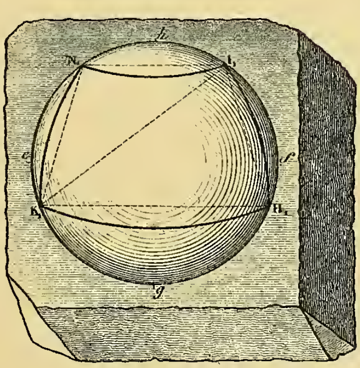

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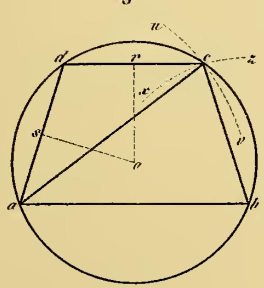


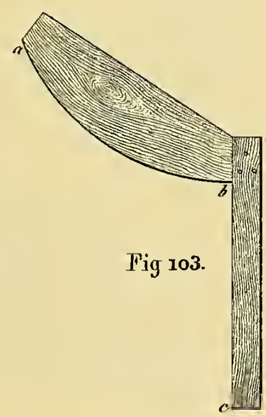
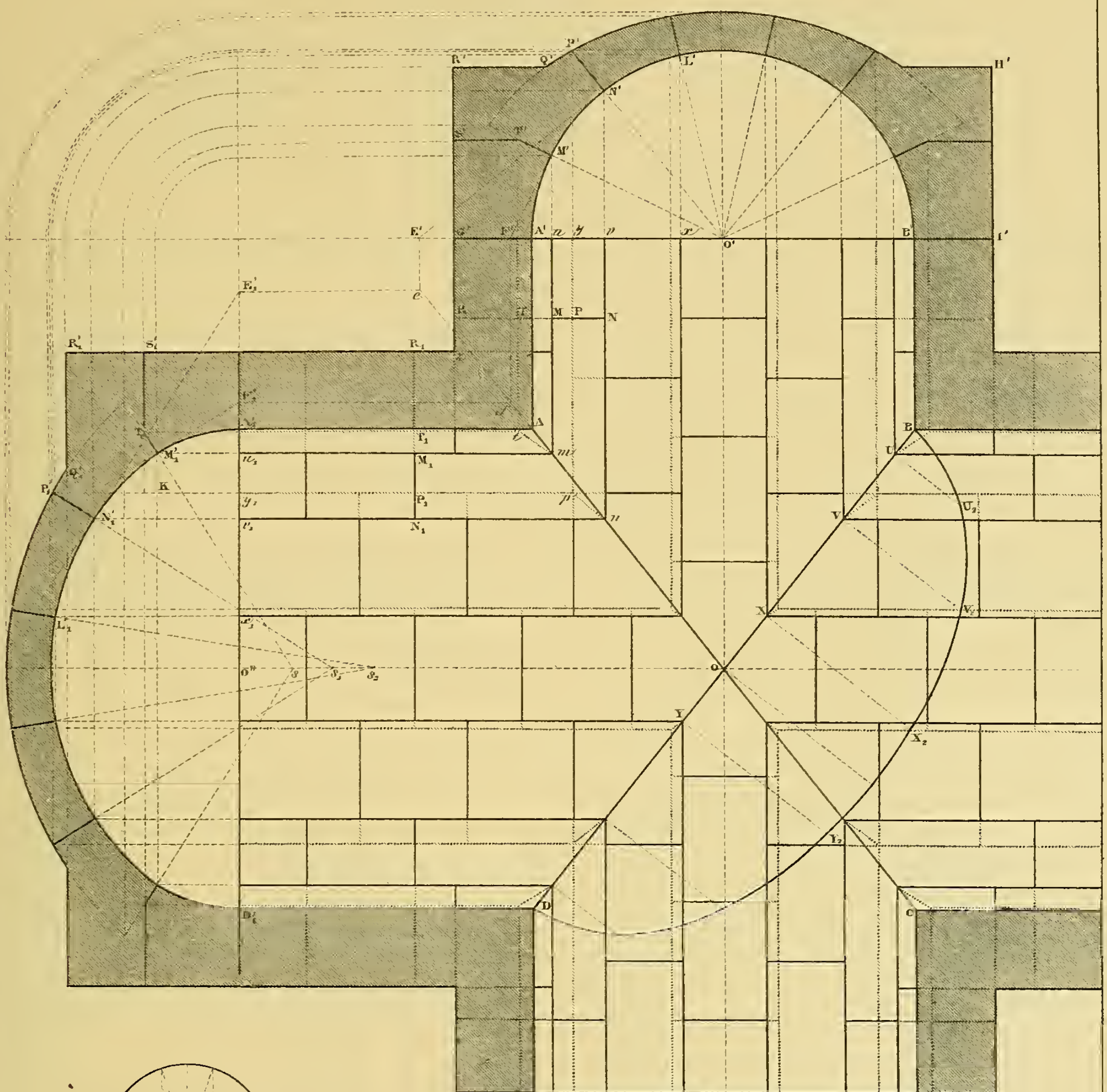


Fig 103.

Em. Lejeune, del.

Fig 104.



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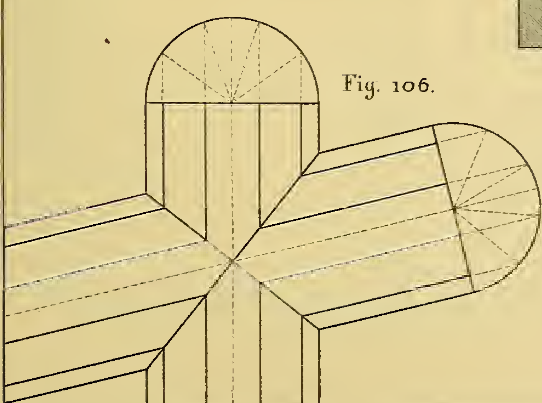

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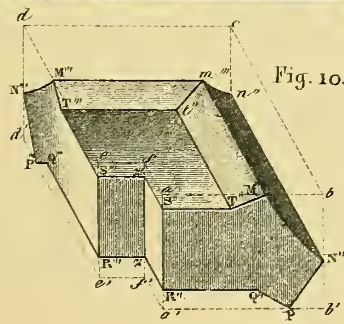
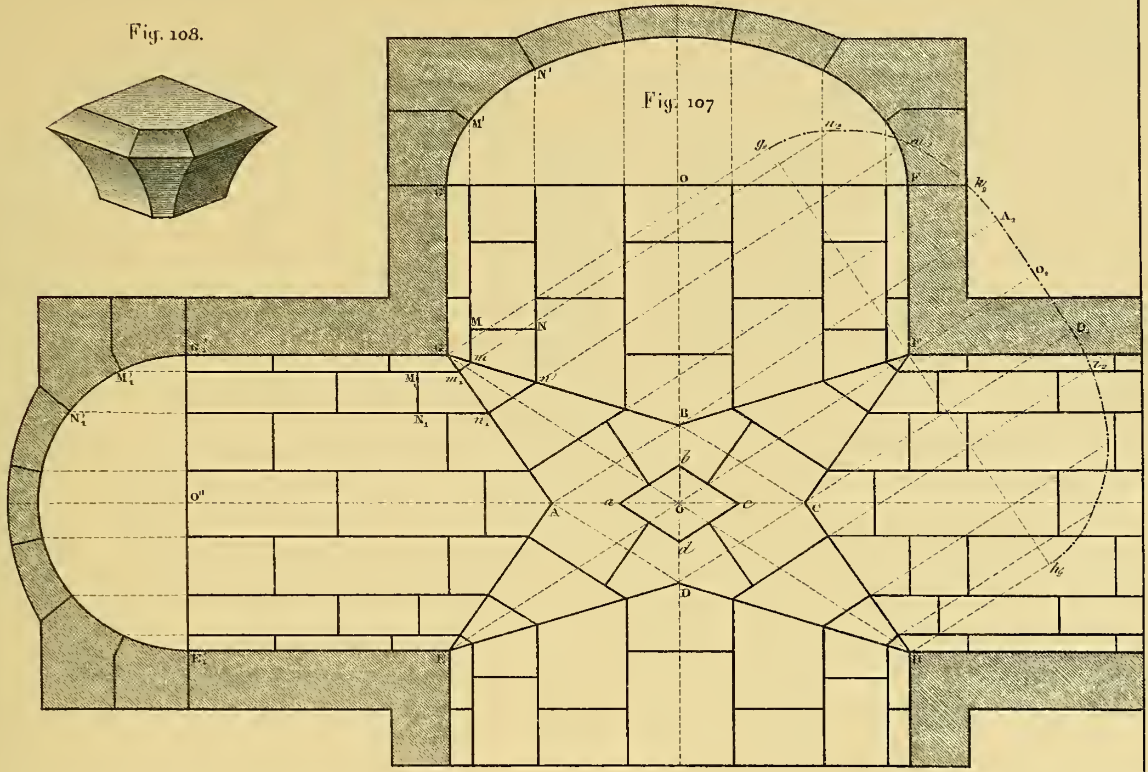


Em. Lejeune del.

Fig. 108.

Fig. 107



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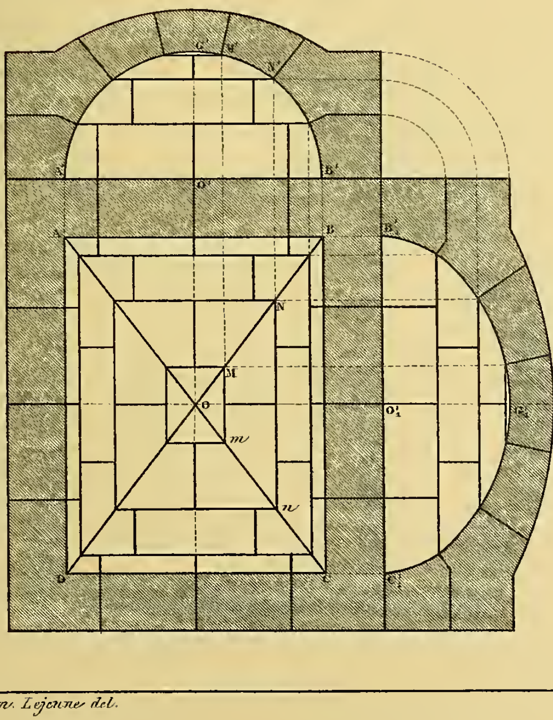

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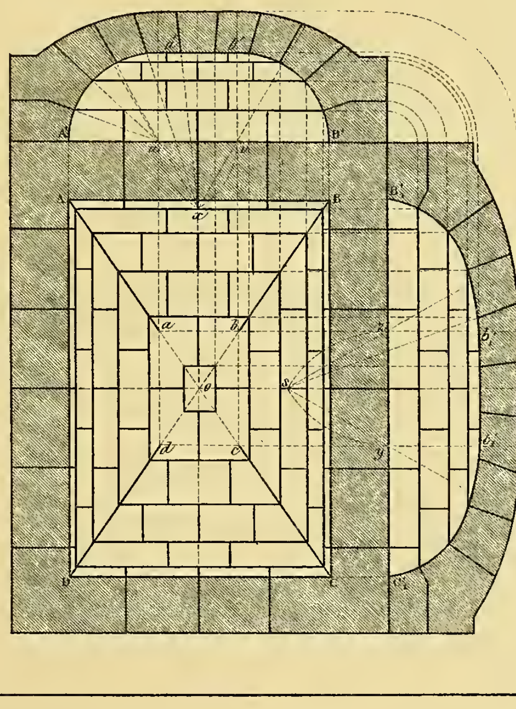

Em. Lejeune del.

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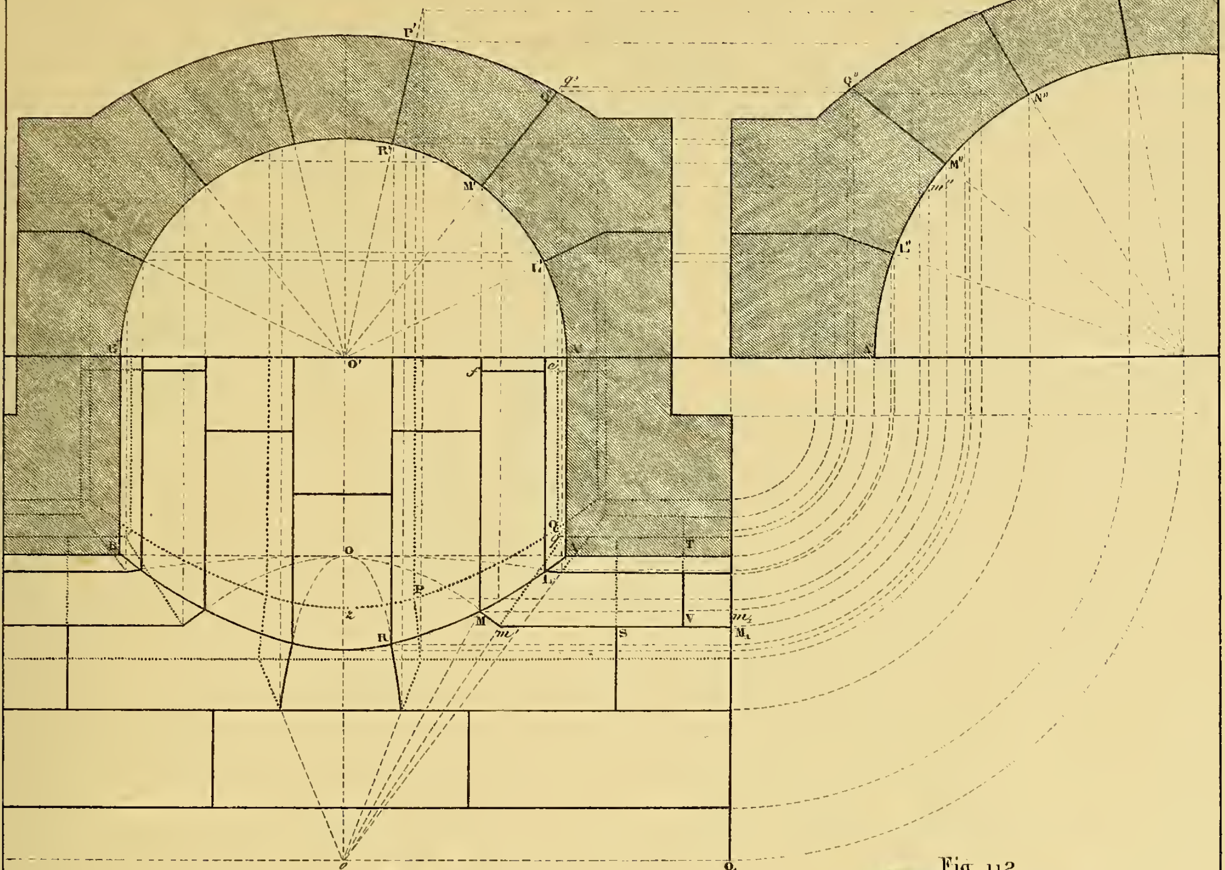

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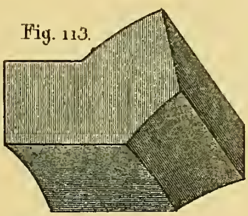

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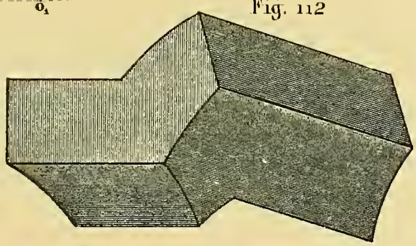

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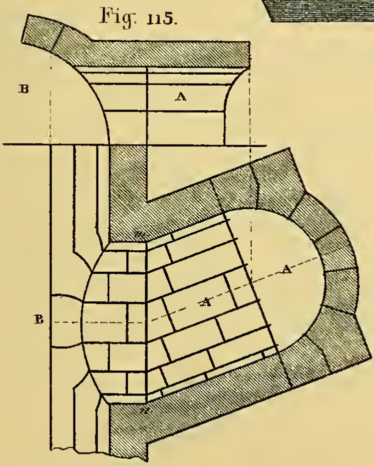

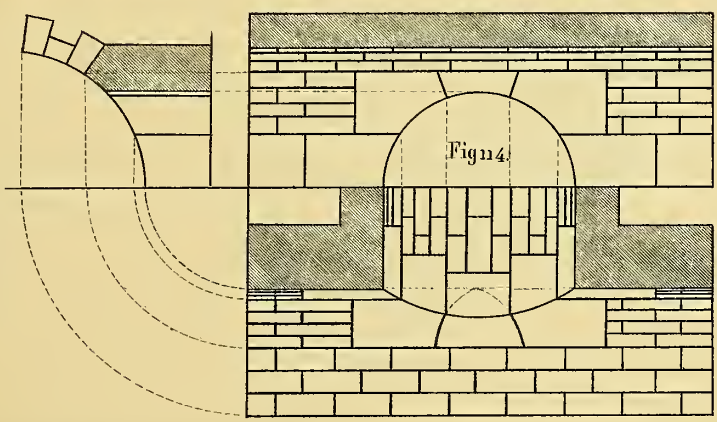

Em. Lejeune del.

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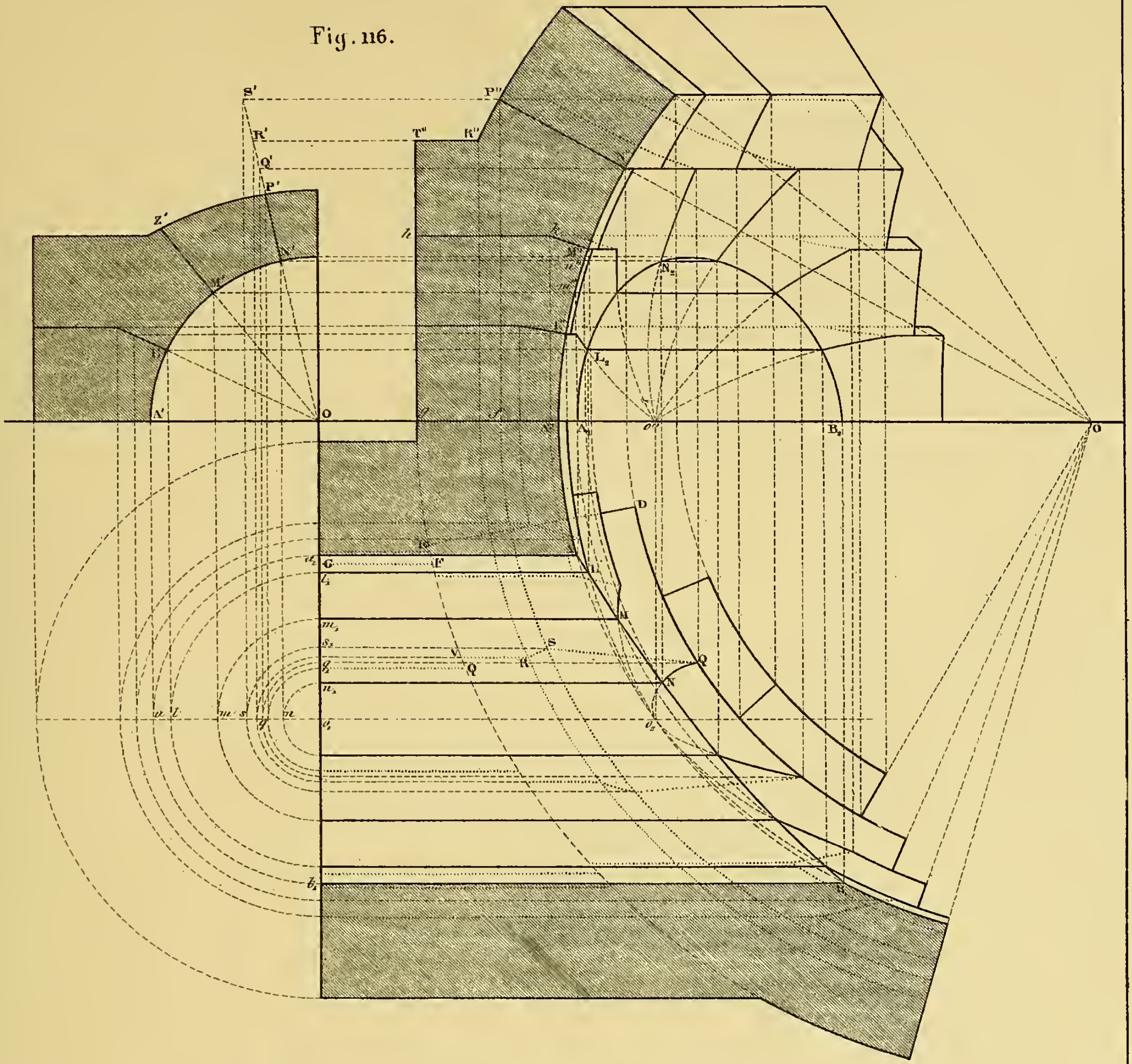

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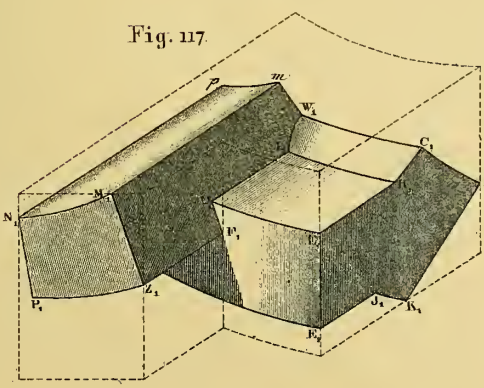
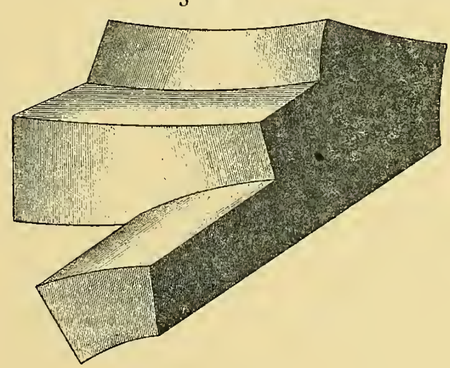


Fig. 118.



Em. Lejeune del.